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# HASM quantum machine learning

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**Abstract** The miniaturization of transistors led to advances in computers mainly to speed up their computation. Such miniaturization has approached its fundamental limits. However, many practices require better computational resources than the capabilities of existing computers. Fortunately, the development of quantum computing brings light to solve this problem. We briefly review the history of quantum computing and highlight some of its advanced achievements. Based on current studies, the Quantum Computing Advantage (QCA) seems indisputable. The challenge is how to actualize the practical quantum advantage (PQA). It is clear that machine learning can help with this task. The method used for high accuracy surface modelling (HASM) incorporates reinforced machine learning. It can be transformed into a large sparse linear system and combined with the Harrow-Hassidim-Lloyd (HHL) quantum algorithm to support quantum machine learning. HASM has been successfully used with classical computers to conduct spatial interpolation, upscaling, downscaling, data fusion and model-data assimilation of eco-environmental surfaces. Furthermore, a training experiment on a supercomputer indicates that our HASM-HHL quantum computing approach has a similar accuracy to classical HASM and can realize exponential acceleration over the classical algorithms. A universal platform for hybrid classical-quantum computing would be an obvious next step along with further work to improve the approach because of the many known limitations of the HHL algorithm. In addition, HASM quantum machine learning might be improved by: (1) considerably reducing the number of gates required for operating HASM-HHL; (2) evaluating cost and benchmark problems of quantum machine learning; (3) comparing the performance of the quantum and classical algorithms to clarify their advantages and disadvantages in terms of accuracy and computational speed; and (4) the algorithms would be added to a cloud platform to support applications and gather active feedback from users of the algorithms.

**Keywords** Quantum computing, Machine learning, Eco-environmental surface, High accuracy surface modelling, Quantum computational advantage, Practical quantum advantage

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## 1 Introduction

Both classical personal computers and supercomputers are based on binary bits in a state of either 0 or 1. But a qubit could be a 0, a 1, or a superposition of both states. The quantum phenomena used for computation is known as quantum computing. The qubits serve as the basic elements of quantum computing and the goal is to drive the qubits from one state to another in a controlled manner and to combine qubits in ways that lead to entanglement. The entanglement occurs when two or more quantum systems

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cannot be described independently, even when they are separated by distance (Mooij, 2005).

Quantum mechanics provides the basis for understanding quantum computation, which is a mathematical framework for the construction of physical theories. It was developed in the first two decades of the twentieth century and assumed its current form in the late 1920s (Nielsen and Chuang, 2010). In 1928, Hilbert et al. (1928) introduced a simple quantum mechanical model, and Turing (1937) proposed a computing machine a decade later. The Turing machine consists of a machine and an infinite tape that was divided into squares. Each square of the tape may be blank or it may contain one of a finite number of symbols. The finite symbol string is called the tape expression. As the machine scans the tape squares one by one, it can change the tape symbol, print a symbol on an empty square, shift a square left or right, or do nothing (Davis, 1958). In 1980, a quantum mechanical model was constructed for standard Turing machines (Benioff, 1980). These findings make it possible to put quantum computing into practice. Quantum computing is based on quantum phenomena such as superposition and entanglement beyond classical computing (Nimbe et al., 2021).

In 1982, Feynman found that the ability of quantum computing would be far stronger than classical computing to process information, which might generate new high-performance computers. Since the first quantum algorithm was presented using quantum parallelism (Deutsch, 1985), many quantum algorithms have been proposed. For instance, an algorithm for factoring large integers demonstrated the computing power of a quantum computer (Shor, 1994). Quantum random walks were developed as the quantum mechanical analogues of the well-known classical random walks, for which roles have been established in quantum information processing (Farhi and Gutmann, 1998; Kempe, 2003). A quantum algorithm was developed for solving linear systems of equations, which provided an exponential speedup compared with classical alternatives (Harrow et al., 2009). A quantum algorithm was proposed for efficiently determining the quality of a least-squares fit over an exponentially large data set (Wiebe et al., 2012). A quantum support vector machine algorithm was proposed for big data classification and provided an exponential speedup compared to classical algorithms (Rebentrost et al., 2014). Quantum algorithms were employed for the Hopfield network, which showed that an exponentially large network can be stored in a polynomial number of quantum bits (Rebentrost et al., 2018). A quantum approximate optimization algorithm was developed for solving combinatorial optimizations with prospects of quantum speedup on near-term devices (Medvidović and Carleo, 2021).

Moore's law states that the number of transistors on a microprocessor chip will double every two years or so. It was worldwide acknowledged that Moore's law was nearing its end (Waldrop, 2016). The "more than Moore strategy" has further motivated the exploration of quantum algorithms, and quantum machine learning algorithms in particular. For instance, an artificial neural network was trained by using qubits as artificial neurons to develop a quantum algorithm, which was carried out on a classical computer (Chalumuri et al., 2020). A novel quantum algorithm based on the entanglement principle was proposed to solve these new types of problems by integrating entanglement measurement and concurrence with quantum parallelism and interference (Zidan et al., 2021). A classical-quantum hybrid algorithm was presented to compute nuclear structure functions in the high-energy Regge limit of Quantum Chromodynamics (QCD) (Mueller et al., 2021). Several quantum algorithms and machine learning methods have been constructed and used to forecast Gross Domestic Product (GDP) growth and to determine the model with higher accuracy. The results demonstrated that intelligent computing could improve the accuracy of quantum algorithms (Alaminos et al., 2022).

In the present era of quantum computing, one of the major goals is the demonstration of quantum computational advantage (QCA) (Deshpande et al., 2022). Since Feynman (1982) proposed that it is exponentially costly to simulate large quantum systems with classical computers, there have appeared many demonstrations of QCA (Deshpande et al., 2022). For instance, an experiment simulating quantum mechanics on a classical computer, by imagining the machine in some superposition of states at every step of the computation, illuminated that a quantum algorithm for factoring of integers is exponentially faster than the classical algorithm (Shor, 1994). The Grover search algorithm (Grover, 1997) exhibited a provable quadratic speedup over the best possible classical algorithm (Bennett et al., 1997). How to define and measure quantum speedup was illustrated with data from tests run on a D-Wave Two device (Rønnow et al., 2014). Biamonte et al. (2017) showed that small quantum information processors can produce statistical patterns in data mining that are computationally difficult for a classical computer, if efficient quantum algorithms can be found for machine learning. The roots of QCA are genuine quantum superposition, the entanglement of observables, and constructive interference (Khrennikov, 2021).

Gaussian boson sampling (GBS) is a feasible approach to demonstrate QCA. For instance, an experiment using a programmable superconducting processor achieved QCA (or quantum supremacy) in the performance of random quantum circuit sampling (Arute et al., 2019). The experiment demonstrated that a 53-qubit processor generated a million noisy samples in 200 s with about 0.2% fidelity, for which a supercomputer would take 10,000 years. Gaussian boson sampling (GBS) in 200 s, with a Jiuzhang photonic quantum computer, would take 2.5 billion years on the Sunway TaihuLight supercomputer (Zhong et al., 2020). Evidence was provided for the hardness of classically simulating GBS and a new programmable architecture was proposed for demonstrating QCA by means of GBS, which bolstered the evidence for QCA in the experiment of Zhong et al. (2020) (Deshpande et al., 2022). A superconducting quantum computing system, Zuchongzhi 2.1, achieved larger-scale random quantum circuit sampling with 60 qubits and 24 cycles, yielding a Hilbert space dimension of up to $2^{60}$ (Zhu et al., 2022). In other words, the development of quantum computing clearly demonstrates its advantage over classical computers for artificial problems.

The practical quantum advantage (PQA) has been discussed in recent years as well (Daley et al., 2022), which means quantum devices can be used to solve problems of practical interest that are not tractable for traditional supercomputers. To demonstrate a PQA, it is required to show a quantum simulator can generate a reliable solution for a relevant problem beyond the bounds of classical simulation with controlled errors, and to implement techniques to quantify the accuracy of a quantum simulation when it is operating beyond the capabilities of current classical simulations.

## 2 HASM quantum machine learning

A raster expression of a region or one of the eco-environmental components in the region can be abstracted to a mathematical surface (Yue, 2011). Under the conceptual framework of Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services (IPBES), we classified the eco-environmental components in nature (including biodiversity, ecosystem structure and geographical features), nature's contribution to people (such as food, freshwater and remediation of environmental pollution), and driving forces of nature changes (such as climate and land use change, policies and regulations) (IPBES, 2016). An eco-environmental surface is a unified term, used to represent a surface of nature, nature's contribution to people, or a driving force of natural changes. Surface modelling is a process of constructing a surface model for dynamically describing the Earth's surface system or a specific component of the Earth's surface environment.

Many studies are based on either satellite observations (Srivastava et al., 2014) or ground observations (Alkhasawneh et al., 2014; Liu et al., 2013). It has been proven that simulation results would have bigger errors when only satellite measurements or ground measurements are considered. For instance, during the last decade, three global land cover datasets, GLC-2000, MODIS and GlobCover, have been derived from satellite remotely sensed images, but there are large spatial discrepancies between these three products and one of the reasons for these discrepancies is the lack of sufficient in-situ data for the development of these products (Fritz et al., 2012). No matter what kind of support vector machine models or parameter optimization methods were used, the relative errors of many points exceeded 20% when predicting mining subsidence in the water area only using ground observation data without satellite observation data in Eastern China (Li et al., 2014). A comprehensive survey in Guyana indicated that estimates only based on remotely sensed data are inaccurate as well (Butt et al., 2015).

Intrinsic and extrinsic information are two essential concepts on which HASM depends. The terms used to describe them are quite different. For instance, mathematicians term them the first and the second fundamental coefficients. Geographers describe them as information on micro-processes and macro-patterns. Ecologists express them as local and global information. Community scientists call them interspecific species-to-species interactions and the surrounding environment (Table 1).

Many investigations have shown that intrinsic and extrinsic information provide complementary streams of information and that neither provides the complete picture (Yue et al., 2020). An eco-environmental surface is controlled by a combination of intrinsic and extrinsic information, which cannot be understood without accounting for both of them (Phillips, 2002; Ponsar et al., 2016). For instance, ground observations can provide high-accuracy data at observation points, but observations at fixed positions are confined within some limited dispersal points and are not able to directly calculate relative parameters at the regional scale. Satellite remote sensing can frequently supply surface information of geographical and ecological processes, but remote sensing description is not able to directly observe process parameters. Remote-sensing data can generate information about the Earth's surface that is impossible from ground-based studies. The timing and extent of land-cover

**Table 1** Terms used to describe intrinsic and extrinsic information in various research fields

| Field | Intrinsic information | Extrinsic information |
|---|---|---|
| Mathematics | The first fundamental coefficients | The second fundamental coefficients |
| Geography | Information on micro-process | Information on macro-pattern |
| Ecology | Local information | Global information |
| Biology | Interspecific species-to-species interactions | The surrounding environment |

change and the relationship between climate and phenology highlight unique information that is available only from satellite and airborne sensors (Chambers et al., 2007). However, maps derived from satellite observations are patchy and cannot be used reliably as an independent source of information for Earth surface monitoring because of the well-known limitations of satellite retrieval, such as missing data for cloud-covered pixels (Emili et al., 2011).

The most effective use of remote-sensing data is through its fusion with appropriate field investigations. For instance, utilizing a satellite image as secondary information decreased the errors associated with yield monitoring data and allowed better prediction in areas where no reliable yield measurements were available (Dobermann and Ping, 2004). Gross primary production (GPP) and net ecosystem exchange (NEE) were simulated by fusing meteorological data derived from the measurements of existing weather stations, forest volume data derived from a previous investigation, satellite data, flux tower data and other ancillary data, which rendered the simulation more stable and accurate (Chiesi et al., 2011). The integration of local observations and remote sensing products can provide a more complete view of the responses of biodiversity to environmental change and can improve the modeling of ecosystem processes across multiple scales (Pereira et al., 2013). Errors in water vapour estimation were reduced using both satellite observations and ground observations over the Walnut Creek region in the USA (Srivastava et al., 2014). Yang et al. (2018) used variance partitioning analysis to show that the phytoplankton community structure is potentially influenced by both extrinsic effects originating from the surrounding environment and intrinsic effects based on interactions between two phytoplankton species. The extrinsic factors explained 31% of the variation in the phytoplankton community, but there were also intrinsic feedback mechanisms and interspecific interactions that determined community response to changes in extrinsic factors, thereby shaping the phytoplankton community structure. Although satellite remote sensing reveals spatial and temporal dimensions of biological diversity through structural, compositional and functional measurements of ecosystems, it cannot directly quantify many other aspects of biodiversity. The integration of remote sensing with in-situ data is needed to fully understand and preserve biodiversity (Cavender-Bares et al., 2022).

A geometric surface is uniquely defined by the first and the second fundamental coefficients (Somasundaram, 2005). The first fundamental coefficients express the information about the details of the surface that are observed when we stay on the surface. The second fundamental coefficients express the change of the surface observed from outside the surface (Yue et al., 2015a). A global model, to be as accurate as possible, must supplement information from the currently available ground observations (Brill et al., 1991). Climate simulations, when combined with observational data from climate stations, contribute significantly toward our most complete understanding of the inter-related complexities associated with the climate system (Bush et al., 2020). IPBES has highlighted the importance of combining indigenous and local knowledge (ILK), which has long been invisible in global scenarios and models. Therefore, it is increasingly clear that there is a need for new global scenarios for nature (Pereira et al., 2020).

All of the findings above described the essential significance of both extrinsic and intrinsic information, but a challenge is how to combine these two kinds of information (Haber, 2021). In order to provide a solution to the challenge, The method for high accuracy surface modelling (HASM) was developed to solve this problem by organically combining surface theory, system theory and cybernetics (Somasundaram, 2005; Djaferis and Schick, 2000; Hull, 2003). HASM has addressed the error, multiscale and nonlinear problems facing eco-environmental surface modelling (Yue et al., 2004, 2007; Yue, 2011).

### 2.1 Principles and applications of classical HASM methods

If an eco-environmental surface can be expressed as $z=f(x, y)$, then HASM has the following formulation,

$$
\begin{cases}
\min \left\| \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \end{bmatrix} \cdot \mathbf{z}^{(n+1)} - \begin{bmatrix} \mathbf{d}^{(n)} \\ \mathbf{q}^{(n)} \\ \mathbf{p}^{(n)} \end{bmatrix} \right\|, \\
\text{s.t.} \\
\quad \mathbf{S}_1 \cdot \mathbf{z}^{(n+1)} = \mathbf{k}_1 \\
\mathbf{k}_2 \leqslant \mathbf{S}_2 \cdot \mathbf{z}^{(n+1)} \leqslant \mathbf{k}_3
\end{cases}
\tag{1}
$$

where $(x, y)$ refers to the geographical coordinates of a raster; $f(x, y)$ represents the value of an eco-environmental component at $(x, y)$; $\mathbf{z}^{(n+1)}$ is the $(n+1)$-*th* iteration of the surface $\mathbf{z}$; $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are coefficient matrices corresponding to the three master equations of HASM; $\mathbf{d}^{(n)}$, $\mathbf{q}^{(n)}$ and $\mathbf{p}^{(n)}$ are right-hand items of the three master equations of HASM, which are determined by the n*th* iteration of the surface $\mathbf{z}$; $\mathbf{d}^{(0)}$, $\mathbf{q}^{(0)}$ and $\mathbf{p}^{(0)}$ are determined by the initial field $\mathbf{z}^{(0)}$ of the surface $\mathbf{z}$; $\mathbf{S}_1$ and $\mathbf{k}_1$ are matrices of ground-observation locations and vector of ground-observation values respectively; $\mathbf{S}_2$ is a location matrix of inequality constraints; and $\mathbf{k}_2$ and $\mathbf{k}_3$ are vectors of the inequality constraints based on prior knowledge.

Intelligent computing has progressed dramatically over the past three decades and is one of today's most rapidly growing technical fields. Intelligent computing (IC), machine learning (ML), and deep learning (DL) are three concepts that have been used frequently together (Abigail and Downs, 2021). IC is the theory by which computers can perform tasks that normally require human intelligence. ML is a subfield within IC and refers to the process of teaching a computer to perform a particular task by improving its performance with experience. DL is a specific type of machine learning and aims to learn the relationships among variables and the knowledge governing the relationships. ML methods can be generally classified into supervised, unsupervised, and reinforcement learning as well as mixtures of them. HASM is a reinforcement learning method. It uses an optimization algorithm to achieve a target surface, $\min\left\|\begin{bmatrix}\mathbf{A}\\\mathbf{B}\\\mathbf{C}\end{bmatrix}\cdot\mathbf{z}^{(n+1)}-\begin{bmatrix}\mathbf{d}^{(n)}\\\mathbf{q}^{(n)}\\\mathbf{p}^{(n)}\end{bmatrix}\right\|$, and a rule for controlling the learning process through ground observations and prior knowledge, $\begin{cases}\mathbf{S}_1\cdot\mathbf{z}^{(n+1)}=\mathbf{k}_1\\\mathbf{k}_2\leqslant\mathbf{S}_2\cdot\mathbf{z}^{(n+1)}\leqslant\mathbf{k}_3\end{cases}$.

HASM has been successfully applied to surface modelling of a wide range of eco-environmental components at various spatial and temporal scales. For instance, constructing a digital terrain model (Yue et al., 2010a, 2010b), filling voids on the elevation surface of the Shuttle Radar Topography Mission (SRTM) (Yue et al., 2012), filling voids on remotely sensed $XCO_2$ surfaces (Yue et al., 2015b), simulating climate change (Yue et al., 2013a, 2016b, 2019), estimating carbon stocks (Yue et al., 2016c), mapping surface soil properties (Shi et al., 2011) and soil pollution (Shi et al., 2009), and analyzing ecosystem responses to climatic change (Yue et al., 2015c). In all these applications, HASM produced more accurate results than the classical methods.

A fundamental theorem for earth surface system modelling (FTESM) (Yue et al., 2016a) and a fundamental theorem for eco-environmental surface modelling (FTEEM) (Yue et al., 2020) were proposed one after another based on developing HASM methods and their successful applications. The FTESM focuses on the Earth's surface system or a component of the Earth's surface environment, which is uniquely defined by both extrinsic and intrinsic properties of the surface, and can be simulated with an appropriate method for integrating the extrinsic and intrinsic properties, such as HASM, when the spatial resolution of the surface is sufficient to capture the attribute(s) of interest. In other words, accurate modelling of the Earth's surface systems requires both extrinsic and intrinsic information as well as appropriate methods. The FTEEM focuses on an eco-environmental surface and is uniquely defined by both extrinsic and intrinsic information of the surface, which can be simulated with an appropriate method for integrating the global and local information, such as HASM, when the spatial resolution of the surface is fine enough to capture the attribute(s) of interest.

Many algorithms have been derived from FTESM and FTEEM (Yue et al., 2020). For instance, HASM algorithms for spatial interpolation were developed to construct a continuous surface from the discrete data or a surface with missing data. HASM algorithms for upscaling were created for the transfer of knowledge from a finer resolution to a coarser resolution to reduce computational costs. HASM approaches to downscaling were developed to obtain information at finer resolution from coarser resolution models and data because spatial resolutions of many models or data are sometimes too coarse to be used for analyses at regional or local scales. HASM methods for data fusion were proposed to integrate multiple data and knowledge streams, which represent the same real-world object, into a consistent, accurate, and useful representation to improve the quality of the information so that it is more accurate than the one in which the data sources were used individually. HASM algorithms for model–data assimilation (MDA) have been used to improve the performance of a model by either optimizing parameters or initial state variables according to ground observations.

Although the error and multi-scale problems were solved by HASM, it could only be used with small areas because it must use the master equation set for simulating each lattice of a surface, which incurs a huge computation cost. To speed up the computation of HASM, several algorithms were developed, such as a multi-grid algorithm of HASM (HASM-MG) (Yue and Song, 2008; Yue et al., 2013b), an adaptive method of HASM (HASM-AM) (Yue et al., 2010a), an adjustment computation of HASM (HASM-AC) (Yue and Wang, 2010), and a preconditioned conjugate gradient algorithm of HASM (HASM-PCG) (Yue et al., 2010b). The multigrid method is the fastest numerical method for solving partial differential equations, which is based on error smoothing and coarse grid correction. The adaptive method marks grid cells where the error is large for refinement and leaves the grid cells with satisfactory accuracy unchanged. The adjustment computation method permits all observations, regardless of their number or type, to be entered into the adjustment and used simultaneously in the computations by means of least squares. A conjugate gradient algorithm was originally viewed as an acceleration technique for the effective solution of large linear systems by a succession of convergent approximations; for example, the preconditioned conjugate gradient algorithm can be developed by introducing a preconditioner to ensure faster convergence of the conjugate gradient method. However, these faster classical algorithms have not solved the slow-speed problem, and hence we have developed quantum algorithms of HASM.

### 2.2 HASM-HHL quantum machine learning

The reinforcement machine learning method of HASM can be transformed into a large sparse linear system. The vectors in the linear system can be replaced by quantum states and eq. (1) can be rewritten as:

$$\mathbf{W}\left|z^{(n+1)}\right\rangle = \left|r^{(n)}\right\rangle, \tag{2}$$

where $\mathbf{W} = \left(\mathbf{A}^T\mathbf{A} + \mathbf{B}^T\mathbf{B} + \beta\mathbf{S}_1^T\mathbf{S}_1\right)$ and $\left|r^{(n)}\right\rangle = \mathbf{A}^T\left|d^{(n)}\right\rangle + \mathbf{B}^T\left|q^{(n)}\right\rangle + \beta\mathbf{S}_1^T\left|k_1\right\rangle$.

In eco-environmental surface modelling, the sparse matrix **W** is usually a huge matrix which is the primary cause of the long computational time when a classical computer is used to run HASM. For a classical computer, among all classical algorithms, the best-performed one takes time $\mathcal{O}(Ns\kappa/\log(\epsilon))$ to solve $\mathbf{A}\cdot\mathbf{x}=\mathbf{b}$ with error $\epsilon$ for an $s$-sparse $N\times N$ matrix **A** with $\kappa$ condition numbers. But using the Harrow-Hassidim-Lloyd (HHL) quantum algorithm to solve the same matrix equation, the operation speed can be improved exponentially, which means the runtime is reduced to $\mathcal{O}(\log(N)s^2\kappa^2/\epsilon)$ (Harrow et al., 2009). Comparing the runtime expressions of the two algorithms, it is obvious that the benefit of the HHL quantum algorithm in shortening the runtime tends to be more significant when the matrix size $N$ becomes larger. Accordingly, we combined HASM machine learning with the HHL quantum algorithm to establish the HASM-HHL algorithm for quantum machine learning. Ideally, HASM-HHL can maintain the high accuracy of classical algorithms and simultaneously achieve exponential speedup compared to the classical algorithms (Yue et al., 2022).

## 3 Discussion

When the HHL quantum algorithm is combined with HASM, it should be noted that the HHL quantum algorithm has certain limitations due to the properties accompanying the quantum phase estimation (QPE). Using QPE to get the eigenvalue $\lambda$ of the matrix **A** is one of the key steps of the HHL quantum algorithm, and in this process, it is necessary to find a suitable parameter $t$ such that $\lambda t\in[0,2\pi]$ (Morrell and Wong, 2021). The selection of the parameter $t$ is directly related to the size of the final calculation error, and a large parameter $t$ will always lead to an obvious increase in calculation time. But in the process of quantum computing, it is more like a black box where we often do not know how this parameter $t$ is selected, so it is impossible to control the resulting errors and calculation time (Shao, 2018). In addition, how to reduce the depth of quantum circuits used in operations is also a key issue that must be faced when using HHL quantum algorithms to solve large-scale matrix equations on real machines (Bravo-Prieto et al., 2019). The hybrid HHL algorithm (h-HHL) formed by combining the HHL quantum algorithm with the classical matrix operation has proved to be a relatively effective method, which can solve the $32\times32$ matrix equation on the Rigetti computer (Bravo-Prieto et al., 2019). This algorithm has also been run on IBM quantum computers and is able to solve 217-dimensional linear equations (Perelshtein et al., 2022). The limitations of the HHL quantum algorithm discussed above will become the problems that need to be considered and solved in the process of optimizing HASM quantum machine learning. At the same time, the h-HHL quantum algorithm also provides good ideas for the subsequent optimization of HASM quantum machine learning.

HASM-HHL would have practical quantum advantages for the future when large-scale quantum computers exist with enough qubits for quantum error correction. However, the capabilities of quantum computers in the near future will continue to be constrained by limitations such as the number of qubits and the effects of noise (Bennewitz et al., 2022). The effect of noise on quantum systems was thought to pose severe obstacles towards the experimental realization of the exponential speed-ups promised by quantum computers over classical computers in the early stage of quantum computation (Aaronson, 2015). In other words, whilst it is clear that quantum computers are much faster than the fastest conventional supercomputers at certain tasks, a fault-tolerant architecture is required to identify and correct errors in noisy quantum hardware.

To find a solution to error correction, Bacon (2006) presented two subsystems of operator quantum error correction (OQEC) to remove the need for a quantum microarchitecture performing quantum error correction and to offer a self-correcting quantum memory. The OQEC is a unified approach to error correction for noise represented by a completely positive trace-preserving linear map that is generally a function of time. Mandayam and Ng (2012) tried to develop a unifying framework for approximate quantum error correction. Cohen et al. (2022) coupled an ancillary system to a quantum error-correcting code in a fault-tolerant way. Bravo-Prieto et al. (2019) presented a hybrid quantum-classical algorithm for solving quantum linear systems, called the variational quantum linear solver (VQLS), by which quantum circuit depth was reduced and noisy intermediate-scale quantum computers could be operated. Bennewitz et al. (2022) introduced a method for neural error mitigation and found that the method provided accurate estimates of more-complex observables without requiring additional quantum resources.

It has been proven that machine learning provides an alternative route to quantum error mitigation and quantum computing speed-up (Caro et al., 2022; Yue et al., 2022). Fawzi et al. (2022) reported a framework for deep reinforcement learning, AlphaTensor, to discover efficient algorithms for the multiplication of arbitrary matrices, which was built on the AlphaZero algorithm that can achieve superhuman performance in many challenging games (Silver et al., 2018). AlphaTensor was operated on a graphics processing unit (GPU) and tensor processing unit (TPU) and made multiplying large matrices 23.9% faster than human-designed algorithms on the same hardware. This idea is helpful for optimizing HASM quantum reinforcement learning.

HASM quantum machine learning might be improved further by: (1) considerably reducing the number of gates required for operating HASM-HHL through the establishment of a hybrid classical-quantum platform for HASM reinforcement learning; (2) proposing a suitable scheme for solving the large sparse linear system based on training experiments on classical supercomputers to optimize quantum circuits and algorithms; (3) finding solutions to the four fundamental problems of quantum machine learning algorithms, i.e., input, output, costing and benchmark problems; and (4) introducing variational quantum eigen-solvers, quantum walks, quantum search, and quantum approximate optimization algorithm into the development of a new HASM machine learning system.

In the meanwhile, two libraries of classical and quantum algorithms will be built for running interpolation, upscaling, downscaling, data fusion, model-data assimilation, and model coupling on classical computers and quantum computers respectively. The performance of the quantum and classical algorithms will then be compared to clarify their advantages and disadvantages in terms of accuracy and computational speed, and the algorithms would be added to a cloud platform to support applications and gather active feedback from users of the algorithms.

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