

Fusion categories and number theory

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BIMSA | Revised 14 October 2026

Abstract. This talk explores connections between fusion categories and number theory:

- Starting from elementary examples of fusion rings, we explain how cyclotomicity, Galois symmetry and divisibility constrain categorification.
- An infinite Fibonacci family of rank-four simple integral fusion rings illustrates a Diophantine construction.
- We then briefly discuss:
 - the relations among modular fusion categories, the modular group and modular forms;
 - how generalizing the length-finiteness of Egyptian fractions summing to 1 leads to rank-finiteness for modular fusion categories;
 - how Stark’s conjectures link the unitarity of cyclic near-group fusion categories to Hilbert’s twelfth problem.

Running order: 37 minutes of presentation + 8 minutes for questions.

Time	Board	Page
0–4	1. From finite groups to fusion rings	2
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37–45	Questions; earlier questions use the same reserve	—

Use. Copy only the material above the lower rule on pages 2–10. The oral cues accompany it; their reference lines are not spoken. Pages 11–20 are preparation and answers to questions, not extra talk material.

Pacing. Reach the first obstruction at minute 13 and the final three connections at minute 27. To recover time, move the coefficient criterion on Board 5 and the numerical example on Board 6 to the oral explanation. The eight-minute question reserve includes interruptions. Stop by minute 45.

Keep visible. $b_i b_j = \sum_k N_{ij}^k b_k$, $d_i = \text{FPdim}(b_i)$ and $D = \sum_i d_i^2$. Later, $\Delta = \dim(C)$ denotes the categorical global dimension; it need not equal D .

BOARD 1

0–4 MINUTES

From finite groups to fusion rings

WRITE ON THE BOARD

$$B = \{1 = b_0, b_1, \dots, b_{r-1}\}, \quad R = \bigoplus_{i=0}^{r-1} \mathbb{Z}b_i.$$

Finite group	Fusion ring
$gh \in G$	$b_i b_j = \sum_k N_{ij}^k b_k, \quad N_{ij}^k \in \mathbb{Z}_{\geq 0}$
$(gh)k = g(hk)$	$(b_i b_j) b_k = b_i (b_j b_k)$
$1g = g1 = g$	$1b_i = b_i 1 = b_i$
$g \mapsto g^{-1}$	$b_i^* \in B, \quad b_i^{**} = b_i \quad (\text{involution on } B)$
$(gh)^{-1} = h^{-1}g^{-1}$	$(b_i b_j)^* = b_j^* b_i^* \quad (\text{extend } \mathbb{Z}\text{-linearly})$
$gh = 1 \iff h = g^{-1}$	$N_{ij}^0 = \delta_{b_j, b_i^*}$

Rank $r = |B|$. In the group case, each basis product is one basis element.

A product may now be a sum with multiplicities.

BRIEF ORAL CUE

Keep associativity and a unit, but allow products to be sums with multiplicities. Commutativity is not assumed; the distinguished basis is part of the structure.

Transition. Tensor products supply the examples.

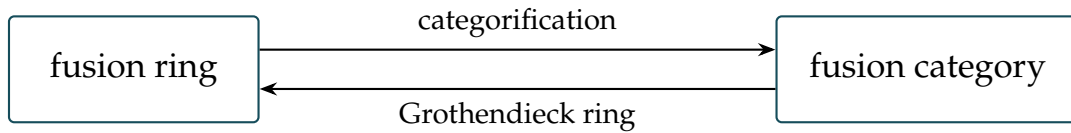
Reference — not spoken. (Etingof–Gelaki–Nikshych–Ostrik, AMS, 2015, Ch. 3). Frobenius reciprocity follows from the displayed axioms.

BOARD 2

4–8 MINUTES

Fusion rings and categorification

WRITE ON THE BOARD



Fusion ring	Fusion category
$\mathbb{Z}[G]$	Vec_G
$\text{ch}(G)$ (character ring)	$\text{Rep}_{\mathbb{C}}(G)$
$B = \{1, x\}, \quad x^2 = 1 + x$	Fib

$$\text{ch}(S_3): \quad \varepsilon^2 = 1, \quad \varepsilon V = V, \quad V^2 = 1 + \varepsilon + V.$$

A categorification of R : a fusion category $\mathcal{C}$ with

$$K_0(\mathcal{C}) \cong R, \quad X_i \otimes X_j \simeq \bigoplus_k X_k^{\oplus N_{ij}^k}.$$

Associativity isomorphisms must satisfy the **pentagon**.

Construction: models / pentagon equations;

Exclusion: arithmetic obstructions.

BRIEF ORAL CUE

A fusion category has finitely many simple types, direct sums, tensor products, duals and finite-dimensional complex morphism spaces. Its unit is simple. Taking the ring forgets the coherence isomorphisms; the opposite arrow is an existence problem, not an inverse function.

Transition. Even dimensions can be irrational.

Reference — not spoken. Here G is finite; Vec_G consists of finite-dimensional G -graded vector spaces. The isomorphism preserves the basis. (Etingof–Gelaki–Nikshych–Ostrik, *AMS*, 2015, Chs. 3–4); (Ostrik, *Math. Res. Lett.*, 2003).

BOARD 3

8–13 MINUTES

Dimensions and a research example

WRITE ON THE BOARD

Frobenius–Perron dimensions (ρ : spectral radius).

$$(N_i)_{kj} = N_{ij}^k, \quad d_i = \text{FPdim}(b_i) = \rho(N_i), \quad d_i d_j = \sum_k N_{ij}^k d_k.$$

$$D = \sum_i d_i^2, \quad R \text{ integral} \iff d_i \in \mathbb{Z} \ (\forall i).$$

$$\text{ch}(G) : d_i = \dim V_i, \ D = |G|; \quad \text{Fib} : d_x = \frac{1 + \sqrt{5}}{2}.$$

Fusion subring: a subring spanned by a duality-stable subset of B containing 1.

Simple: $r > 1$; only fusion subrings are $\mathbb{Z}1$ and R .

Fusion subrings of $\text{ch}(G)$: $\text{ch}(G/N), \ N \trianglelefteq G$.

$$\boxed{\text{ch}(G) \text{ simple} \iff G \text{ simple}} \quad (G \neq 1).$$

My main research motivation:

Does there exist $R = K_0(C)$ simple and integral,
 $R \neq \text{ch}(G)$ for every finite group G ?

BRIEF ORAL CUE

For groups these dimensions are character degrees. Simplicity concerns basis subsets, not ideals. My main research motivation is to find a simple integral categorification beyond finite-group character rings: one illustration of the arithmetic connections in this talk.

Transition. A plausible multiplication table may already fail an arithmetic test.

Reference — not spoken. (Etingof–Gelaki–Nikshych–Ostrik, AMS, 2015); (Liu–Palcoux–Ren, *Internat. J. Math.*, 2023); (Etingof–Nikshych–Ostrik, *Adv. Math.*, 2011). The group-quotient correspondence is proved on page 15. All ring isomorphisms preserve the distinguished basis.

BOARD 4

13–18 MINUTES

Cyclotomicity: a worked exclusion

WRITE ON THE BOARD

Cyclotomicity theorem. (Etingof–Nikshych–Ostrik, *Ann. of Math.*, 2005)

$$R = K_0(C) \implies d_i \in \mathbb{Z}[\zeta_N] \text{ for some } N.$$

Kronecker–Weber: for a number field K ,

$$K \subseteq \mathbb{Q}(\zeta_N) \text{ for some } N \iff K/\mathbb{Q} \text{ is abelian Galois.}$$

(Washington, Springer, 1997, Ch. 14)

A genuine rank-three fusion ring:

$$X^2 = 1 + 2Y, \quad XY = 2X + Y, \quad Y^2 = 1 + X + 2Y.$$

$$N_X = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}, \quad p(t) = t^3 - t^2 - 5t + 1.$$

p is irreducible, and

$$\text{disc}(p) = 16 \cdot 37 \notin (\mathbb{Q}^\times)^2 \implies \text{Gal}(p) = S_3.$$

The splitting field is nonabelian: no categorification over $\mathbb{C}$.

BRIEF ORAL CUE

The discriminant is displayed because its nonsquareness forces S_3 for this irreducible cubic. If one root lay in a cyclotomic field, all its conjugates and hence its splitting field would lie there too, contradicting abelianity.

Transition. The field is only the first obstruction.

Reference — not spoken. (Etingof–Nikshych–Ostrik, *Ann. of Math.*, 2005, Thm. 8.51 and Cor. 8.54); (Ostrik, *Math. Res. Lett.*, 2009, Ex. 1.6). Associativity and the polynomial calculation are checked on page 16.

BOARD 5

18–23 MINUTES

Codegrees: Galois symmetry and divisibility

WRITE ON THE BOARD

Here R is commutative, with ring characters $\chi_j : R \rightarrow \mathbb{C}$. Its **formal codegrees** are

$$f_j = \sum_i |\chi_j(b_i)|^2, \quad f_{\text{FP}} = D, \quad \sum_j \frac{1}{f_j} = 1.$$

Categorification implies: each $f = f_j$ is a d -**number**. (Ostrik, *Math. Res. Lett.*, 2009)

$$f/\sigma(f) \in \overline{\mathbb{Z}}^{\times} \quad (\forall \sigma \in \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})), \quad \overline{\mathbb{Z}} = \text{all algebraic integers.}$$

Equivalently, its minimal polynomial satisfies

$$p_f(t) = t^m + a_1 t^{m-1} + \cdots + a_m \in \mathbb{Z}[t] \implies a_m^i \mid a_i^m \quad (1 \leq i \leq m).$$

If f has degree 2: $p_f(t) = t^2 - at + b$, so $b \mid a^2$.

General divisibility theorem. No pivotality assumption:

$$\Delta/f_j \in \overline{\mathbb{Z}}, \quad \Delta = \dim(C).$$

(Ostrik, *Mosc. Math. J.*, 2015, Cor. 2.14)

Integral categorification: $\Delta = D$ and $f_j \in \mathbb{Z}_{>0}$, $f_j \mid D$.

$$\text{ch}(G) : f_j = |C_G(g_j)|; \quad \text{ch}(S_3) : \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1.$$

BRIEF ORAL CUE

A ring character is a simultaneous eigenvalue system; its squared length is a codegree. The d -number condition means that its principal ideal is Galois-invariant. The divisibility theorem uses the categorical global dimension, which agrees with D in the integral case.

Transition. Yet integrality alone leaves infinitely many rings.

Reference — not spoken. The coefficient test is (Ostrik, *Math. Res. Lett.*, 2009, Lem. 2.7). The quadratic polynomial is the minimal polynomial of a codegree, not of an arbitrary basic dimension. General divisibility and its integral strengthening are explained on page 14.

BOARD 6

23–27 MINUTES

A Diophantine problem in rank four

WRITE ON THE BOARD

Question. Infinitely many simple integral fusion rings of a fixed rank?

Yes in rank 4. (Dong–Palcoux–Plessis, work in progress, 2026)

$$F_0 = 0, \quad F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}.$$

One infinite family, indexed by $n \geq 1$, has type

$$\left[1, 1 + F_{2n+5}, F_{n+1} + F_{n+3}F_{2n+3}, F_n + F_{n+2}F_{2n+3} \right].$$

Type = list of basic dimensions, up to order. For $n = 1$:

$$(d_i) = (1, 11, 14, 16), \quad D = 574.$$

1-Frobenius: $d_i \mid D$ for every i .

No simple integral rank-four fusion ring is 1-Frobenius.

(Dong–Palcoux–Plessis, work in progress, 2026)

Here $11 \nmid 574$. This differs from the codegree condition $f_j \mid D$.

$$\underbrace{d_i \mid D \quad \text{for every integral fusion category}}_{\text{conjectural in general}}.$$

BRIEF ORAL CUE

The Fibonacci types come from a Pell-type equation, not a numerical search. They give infinitely many distinct rings, without claiming to exhaust all rank-four rings. Failure of Frobenius divisibility alone is not a categorical exclusion: the categorical property is conjectural.

Transition. With extra structure, arithmetic controls entire families.

Reference — not spoken. (Dong–Palcoux–Plessis, work in progress, 2026); for Frobenius type and the proved weakly group-theoretical case, (Etingof–Nikshych–Ostrik, *Adv. Math.*, 2011). The displayed example is independently excluded by its noninteger codegrees; see page 15.

BOARD 7

27–30 MINUTES

Modular categories and modular forms

WRITE ON THE BOARD

Modular: braided, ribbon, nondegenerate; modular data (S, T) give a projective representation of $\mathrm{SL}_2(\mathbb{Z})$.

$$\chi_j(b_i) = \frac{S_{ij}}{S_{0j}}, \quad T = \mathrm{diag}(\theta_i).$$

S : normalized character table. θ_i : twist phase of a full turn, the *topological spin*.

Theorem. A strongly rational vertex operator algebra V gives a modular category $\mathrm{Rep}(V)$. (Huang, *Commun. Contemp. Math.*, 2008)

VOA-realization conjecture:

Every modular $C \simeq \mathrm{Rep}(V)$ for some such V .

Its graded characters are the modular form:

$$F_i(\tau) = \mathrm{Tr}_{M_i} \left(q^{L_0 - c/24} \right), \quad q = e^{2\pi i \tau}.$$

$\mathbf{F} = (F_i)_i$: vector-valued, weakly holomorphic, **weight** 0.

$$\mathbf{F}(-1/\tau) = S\mathbf{F}(\tau), \quad \mathbf{F}(\tau + 1) = \widehat{T}\mathbf{F}(\tau), \quad \widehat{T} = e^{-2\pi i c/24}T.$$

BRIEF ORAL CUE

See Yilong's talk for more details. Here $\mathrm{Rep}(V)$ means V -modules, and the equivalence preserves tensor products, braiding and twists. M_i is an irreducible module, L_0 its grading operator and c the central charge. Weight zero means no power of τ occurs in the transformation law.

Transition. The same modular categories also lead to Egyptian fractions.

Reference — not spoken. (Zhu, *JAMS*, 1996); (Huang, *Commun. Contemp. Math.*, 2008); (Möller-Rayhaun, *arXiv*, 2024, §2.3). The S -normalization is the VOA one; $\widehat{T}$, not the bare twist matrix T , acts on characters. Precise hypotheses and the allowed cusp poles: page 17.

BOARD 8

30–33 MINUTES

From Egyptian fractions to rank-finiteness

WRITE ON THE BOARD

Start with a modular category of rank r . Write $\delta_i = \dim(X_i)$ and $\Delta = \sum_i \delta_i^2$. Its formal codegrees give

$$f_i = \frac{\Delta}{\delta_i^2} \in \overline{\mathbb{Z}}_{>0}, \quad \boxed{\sum_{i=0}^{r-1} \frac{1}{f_i} = 1.}$$

Integral case: $f_i \in \mathbb{Z}_{>0}$. Fixed length r gives finitely many Egyptian fractions summing to 1, hence rank-finiteness in this case.

General case: algebraic denominators with controlled primes. Put $N = \text{ord}(T)$ and $K = \mathbb{Q}(\zeta_N)$; rank bounds N .

An **$\mathcal{S}$ -unit** here is $u \in K^\times$ such that *both u and u^{-1} are algebraic integers divided by powers of N* . $\mathcal{S}$ records the allowed primes, not Stark. Each $u_i = 1/f_i$ is such a unit.

Finiteness theorem. For fixed K, N, r ,

$$u_i > 0 \text{ are } \mathcal{S}\text{-units, } \sum_i u_i = 1 \implies \text{finitely many tuples.}$$

(Evertse, *Compos. Math.*, 1984)

In both cases, finitely many (f_i) bound $\text{FPdim}(C) = f_{\text{FP}}$; rigidity gives

Fixed rank: finitely many modular fusion categories
up to equivalence.

(Bruillard–Ng–Rowell–Wang, *JAMS*, 2016)

BRIEF ORAL CUE

The length of the sum is the rank. Integer denominators give an elementary finiteness theorem; repetitions are allowed. For algebraic denominators we need control of the field and the primes. Over $\mathbb{Q}$, allowing 2 and 3 gives $\pm 2^a 3^b$, with $a, b \in \mathbb{Z}$. Over a number field, both a number and its inverse must have only the allowed primes in their denominators. Modularity supplies that control, and Evertse gives finiteness.

Transition. Here $\mathcal{S}$ is a prime set, not Stark. The next board uses special units for unitarity, not an assumption needed for rank-finiteness.

Reference — not spoken. Codegrees: (Ostrik, *Mosc. Math. J.*, 2015, Ex. 2.9). Positive solutions automatically have no vanishing subsum, so (Evertse, *Compos. Math.*, 1984) applies. The prime control, rank bound and rigidity are in (Bruillard–Ng–Rowell–Wang, *JAMS*, 2016, Thm. 3.9, Prop. 3.15, Cor. 3.13). The definition used here takes all primes above N ; details: page 11.

BOARD 9

33–37 MINUTES

Near-group unitarity and Stark's conjecture

WRITE ON THE BOARD

Our theorem. Every near-group fusion ring $C_n + n$, $n \geq 1$, has a **pseudo-unitary categorification**.

$$g\rho = \rho g = \rho, \quad \rho^2 = \sum_{g \in C_n} g + n\rho.$$

(He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026); see also (Radchenko–Wheeler, arXiv, Sept. 2026).

The associativity equations are solved. **Unitarity** additionally requires compatible positive-definite inner products.

The remaining check. In our construction, certain algebraic parameters z_k , $1 \leq k < n$, must be **phases**: $|z_k| = 1$.

Why Stark? The z_k are conjugates of explicit special-function values related to Stark units. The conjecture controls their Galois conjugates; in our positive-dimension realization it gives

$$\text{Stark} \implies \overline{z_k} = z_k^{-1} \iff |z_k| = 1 \implies \text{unitarity}.$$

Hilbert 12. Find explicit special-function generators of abelian extensions, generalizing the role of roots of unity over $\mathbb{Q}$. Stark connects these same values to this problem for real quadratic fields.

Unitarity is proved for $1 \leq n \leq 6$ without Stark; the general implication is conditional.

BRIEF ORAL CUE

We have categorified every $C_n + n$ pseudo-unitarily. What remains is compatibility with inner products. In our formulas, this reduces to checking that certain numbers are phases. The algebraic equations already pair these parameters by inversion; unitarity asks that inversion be complex conjugation. These numbers are special-function values also occurring in Stark's conjectures. Stark predicts their Galois behavior and, in our positive-dimension realization, supplies exactly this missing condition. Hilbert's twelfth problem asks for explicit special values generating abelian extensions. Stark links the same numbers to this problem over real quadratic fields.

Transition. The same special values connect categorical unitarity with explicit number-field constructions.

Reference — not spoken. Stark is sufficient, not known to be necessary. The precise conjecture is the order-one abelian Stark conjecture for real quadratic fields. (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026, Cor. 6.6, Props. 6.7 and 6.9, Conj. 6.8, Rem. 6.11); (Appleby–Flammia–Kopp, arXiv, 2025, Thm. 2.20). Definitions and details: pages 12–13.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Board 8 unpacked: fractions, units and finiteness

The modular equation comes first. For a modular category of rank r , set $\delta_i = \dim(X_i)$ and $\Delta = \sum_i \delta_i^2$. The formal codegrees satisfy

$$f_i = \Delta / \delta_i^2 \in \overline{\mathbb{Z}}_{>0}, \quad u_i = 1/f_i = \delta_i^2 / \Delta > 0, \quad \sum_{i=0}^{r-1} u_i = 1.$$

Thus we sum the *reciprocals* of the codegrees, not the codegrees themselves. The unit column gives $f_0 = \Delta$, whereas the Frobenius–Perron character gives $f_{\text{FP}} = D = \text{FPdim}(\mathcal{C})$; these need not be the same column. See (Ostrik, Mosc. Math. J., 2015, Ex. 2.9 and Rem. 2.12).

Integral case: ordinary Egyptian fractions. By Board 5, all f_i are positive integers. Repeated denominators are allowed: a pointed rank- r category gives r copies of $1/r$. For fixed r , order the denominators as $a_1 \leq \dots \leq a_r$. Then $1 \leq r/a_1$, so $a_1 \leq r$. After fixing a_1 , apply the same bound to the positive rational remainder and the remaining terms; the last denominator is determined. This proves length-finiteness, without restricting primes. Finitely many tuples bound $D = f_{\text{FP}}$ and hence give rank-finiteness by the categorical step below.

A definition without valuations. Let K be a number field and O_K its algebraic integers. Fix $N \geq 1$ and allow powers of N in denominators. In the case used here,

$$u \text{ is an } \mathcal{S}\text{-unit} \iff u, u^{-1} \in O_K[1/N] \iff N^a u, N^b u^{-1} \in O_K \text{ for some } a, b \in \mathbb{Z}_{\geq 0}.$$

Here $\mathcal{S}$ consists of *all* prime ideals of O_K dividing (N) . This is exactly the special case of (Bruillard–Ng–Rowell–Wang, JAMS, 2016, Def. 3.16) needed for modular categories, not a definition for an arbitrary subset of primes above N . Over $\mathbb{Q}$, $N = 6$ gives precisely $\pm 2^a 3^b$, $a, b \in \mathbb{Z}$. In a number field, ordinary algebraic units are also included: $1 + \sqrt{2}$ and its inverse $\sqrt{2} - 1$ are already algebraic integers. This avoids assuming unique factorization of elements.

Evertse and the modular application. For fixed $K, \mathcal{S}, r$, the equation $\sum_i u_i = 1$ has finitely many $\mathcal{S}$ -unit solutions with no nonempty vanishing subsum (Evertse, Compos. Math., 1984). Positivity ensures this hypothesis. Without it, $(t, -t, 1)$ may give infinitely many solutions. In the modular case $N = \text{ord}(T)$ is bounded by r , and $K = \mathbb{Q}(\zeta_N)$. The categorical Cauchy theorem gives the required prime control for Δ ; the algebraic integrality of Δ / δ_i^2 gives it for δ_i as well. Hence f_i and u_i are $\mathcal{S}$ -units. There are only finitely many possible $(K, \mathcal{S})$, so Evertse gives finitely many codegree tuples (Bruillard–Ng–Rowell–Wang, JAMS, 2016, Thm. 3.9, Prop. 3.15 and §3.2).

Why number finiteness implies category finiteness. In either case $D = f_{\text{FP}}$ is bounded. Since $N_{ij}^k d_k \leq d_i d_j$ and $1 \leq d_i \leq \sqrt{D}$, the fusion coefficients are bounded. At fixed rank there are finitely many tables; rigidity gives finitely many modular categorifications (Bruillard–Ng–Rowell–Wang, JAMS, 2016, Cor. 3.13).

Two safeguards. Unrestricted algebraic integers do not suffice: for $v = (3 + 2\sqrt{2})^m$, $m \geq 1$, the totally positive algebraic integers $f = 1 + v$, $g = 1 + v^{-1}$ satisfy $1/f + 1/g = 1$, giving infinitely many pairs in one field. Also, $\mathcal{S}$ is a prime set, *not Stark*: Board 8 uses a proved finiteness theorem; Board 9 uses conjectural special-value and Galois information. A Stark unit may be an $\mathcal{S}$ -unit, but these notions are not synonyms, and being a unit does not mean having modulus one.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Board 9 unpacked: why phases give unitarity

Start with what is already proved. For every $n \geq 1$, the ring $C_n + n$ has a pseudo-unitary categorification. In its canonical spherical structure, the simple dimensions are 1 and

$$d_+ = \frac{n + \sqrt{n(n+4)}}{2} = \text{FPdim}(\rho).$$

This is a theorem without Stark. It does not yet provide compatible positive-definite inner products on morphism spaces. See (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026, Cor. 6.6); the independent construction (Radchenko–Wheeler, arXiv, Sept. 2026) remains credited.

Why solving the equations does not already give unitarity. An associativity matrix can be invertible without being unitary. The pentagon imposes polynomial identities; unitarity additionally requires inverse maps to be adjoints for suitable inner products. Algebraic Galois transport preserves the polynomial identities, but need not preserve ordinary complex conjugation. This is precisely the distinction in (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026, Rem. 6.10).

The scalar version of that distinction. For the particular parameters in this construction, the algebraic equations already give

$$z_{n-k} = z_k^{-1}.$$

The remaining Hermitian condition asks that the same partner be the complex conjugate:

$$z_{n-k} = \overline{z_k}.$$

Together these mean $z_k \overline{z_k} = 1$, or $|z_k| = 1$. For these *specified* parameters, the reconstruction theorem turns this scalar condition into a unitary tensor structure. It is not a general claim that an associativity matrix is unitary whenever its entries have modulus one. See (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026, Prop. 6.7).

Exactly which numbers are the z_k ? The analytic solution first has dimension $d_- = (n - \sqrt{n(n+4)})/2 < 0$. Let L_n be its finite coefficient field and choose *one* embedding $\sigma : L_n \hookrightarrow \mathbb{C}$ with $\sigma(d_-) = d_+$. If $u_k = F(k)$ denotes the hyperbolic-gamma value in equation (2.20) of the preprint, set

$$z_k = \sigma(u_k), \quad 1 \leq k < n.$$

The identity $u_{n-k} = u_k^{-1}$ survives under σ . The same embedding acts on every coefficient; choosing conjugates independently need not preserve the equations. The notation u_k avoids the modular-form vector $\mathbf{F}$ on Board 7. See (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026, Props. 6.5 and 6.7).

What remains conditional? The order-one abelian Stark conjecture implies $|z_k| = 1$ for all k under every such embedding; one common embedding is enough for a unitary construction. This is the implication after Conjecture 6.8 of (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026), using (Appleby–Flammia–Kopp, arXiv, 2025, Thm. 2.20(2)). For $1 \leq n \leq 6$, the condition is proved without Stark (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026, Prop. 6.9). Thus Stark is a sufficient route at arbitrary order, not a proved necessary assumption. The board does not assert that all pseudo-unitary fusion categories are unitary.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Board 9 unpacked: Stark and Hilbert 12

The intuition needed for the talk. The construction supplies special algebraic numbers, but unitarity depends on how their chosen complex conjugates behave. Stark’s conjectures predict arithmetic properties of special-function values, including information about their Galois conjugates. Our parameters are related to those values. In the realization with positive dimensions, the relevant prediction is exactly

$$\overline{z_k} = z_k^{-1} \iff |z_k| = 1.$$

This explains the role of Stark without defining a zeta function on the board. The result used is (Appleby–Flammia–Kopp, [arXiv, 2025](#), Thm. 2.20(2)), specialized as in (He–Liu–Lu–Palcoux–Ren, [arXiv, Oct. 2026](#), Prop. 6.4 and discussion after Conj. 6.8).

Why the word “unit” alone would not explain it. An algebraic unit is an algebraic integer whose inverse is also an algebraic integer. For example $1 + \sqrt{2}$ is a unit, although its absolute value is not 1. The $\mathcal{S}$ in Board 8 denotes allowed primes, not Stark; that finiteness theorem assumes no Stark conjecture. The needed input is the predicted behavior under the *relevant embeddings*, not just algebraicity or invertibility. The original analytic values themselves are not all being asserted to lie on the unit circle.

Hilbert 12 in one sentence. Find explicit special-function values that generate abelian extensions of number fields. “Abelian” means that the extension is Galois with commutative Galois group. Over $\mathbb{Q}$, the model is cyclotomic: every finite abelian extension is contained in a field generated by a root of unity, not necessarily equal to that whole cyclotomic field. See (Washington, Springer, 1997, Ch. 14).

For our family, the underlying real quadratic field is

$$K_n = \mathbb{Q}\left(\sqrt{n(n+4)}\right).$$

Stark connects our special values both to abelian extensions of K_n and to the Galois behavior that supplies unitarity. These are two roles of the *same* analytic values, not two unrelated appearances of number theory. This is the connection explained in (He–Liu–Lu–Palcoux–Ren, [arXiv, Oct. 2026](#), Rem. 6.11).

Optional arithmetic detail — only for questions. The parameters are, up to roots of unity and inversion, real-multiplication values of the Shintani–Faddeev modular cocycle. Kopp’s limit formula relates squares of these values to exponentials of derivatives at $s = 0$ of differences of partial zeta functions. Such a partial zeta function is an ideal-counting series restricted to a specified ray class, continued analytically before differentiation. Stark predicts that the resulting analytic data describe special algebraic units and their conjugate absolute values. See (He–Liu–Lu–Palcoux–Ren, [arXiv, Oct. 2026](#), Rem. 6.11); (Appleby–Flammia–Kopp, [arXiv, 2025](#), §2.6 and Thm. 2.20).

Under the order-one Stark conjecture, a *positive power* of the relevant value lies in an abelian extension, and the root-reversing conjugates have modulus one. Finer placement of the value itself can require stronger assumptions. We do not claim that our cyclic family generates every abelian extension or solves Hilbert 12. Algebraicity of the coefficient field is already unconditional (He–Liu–Lu–Palcoux–Ren, [arXiv, Oct. 2026](#), Prop. 6.5); the missing ingredient for unitarity is the conjugate-modulus statement.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Board 5 unpacked: codegree tests without hidden hypotheses

The coefficient criterion is an equivalence. For a nonzero algebraic integer f of degree m , write

$$p_f(t) = t^m + a_1 t^{m-1} + \cdots + a_m \in \mathbb{Z}[t].$$

Then f is a d -number if and only if $a_m^i \mid a_i^m$ for all $1 \leq i \leq m$. This is (Ostrik, Math. Res. Lett., 2009, Lem. 2.7); for a codegree $a_m \neq 0$, so the divisions are well-defined. The condition at $i = m$ is automatic. The Galois formulation says that the principal ideal of f in $\overline{\mathbb{Z}}$ is invariant, or equivalently that every $f/\sigma(f)$ is an algebraic unit.

For degree two, $p_f(t) = t^2 - at + b$ with $a = f + f'$ and $b = ff'$. The criterion is precisely $b \mid a^2$. Indeed, for $v = f/f'$,

$$v + v^{-1} = \frac{a^2 - 2b}{b}, \quad v v^{-1} = 1.$$

If v is a unit, the displayed rational trace is an integer. Conversely, if $b \mid a^2$, then v and v^{-1} are roots of the monic polynomial $t^2 - (a^2/b - 2)t + 1$, so both are algebraic integers. This is a test on a codegree, not on an arbitrary basic dimension.

Global dimension is defined without a pivotal structure. For an arbitrary fusion category over $\mathbb{C}$, $\Delta = \dim(C)$ is the sum of the squared norms of its simple objects. For a spherical structure it is $\sum_i \delta_i^2$. In general it is not the Frobenius–Perron global dimension $D = \sum_i d_i^2$. The general theorem is

$$\Delta/f_E \in \overline{\mathbb{Z}} \quad \text{for every irreducible ring representation } E.$$

No pivotal, spherical or pseudo-unitary hypothesis is required; the statement also holds for noncommutative Grothendieck rings. See (Ostrik, Mosc. Math. J., 2015, Cor. 2.14).

Why the proof does not require sphericity. For spherical C , the induced unit in its Drinfeld center contains a simple object A_E with $\dim(A_E) = \Delta/f_E$, an algebraic integer. For general C , sphericalization doubles both the global dimension and the codegree of the relevant lifted representation: the same argument gives $(2\Delta)/(2f_E) = \Delta/f_E$. This is the proof of the cited corollary, not an assumption that C itself is spherical.

Why the integral conclusion is stronger. An integral fusion category is pseudo-unitary and has a canonical spherical structure with $\delta_i = d_i$ and $\Delta = D$. In that structure,

$$D/f_E = \dim(A_E) = \dim(F(A_E)) \in \mathbb{Z}_{>0},$$

where $F : \mathcal{Z}(C) \rightarrow C$ is forgetful. Consequently f_E is rational; being an algebraic integer, it is an integer dividing D . This uses the central-object realization, not merely that D/f_E is algebraic integral. See (Etingof–Nikshych–Ostrik, Ann. of Math., 2005, §8.4); (Ostrik, Mosc. Math. J., 2015, Thm. 2.13). This codegree divisibility is proved, whereas $d_i \mid D$ in all integral fusion categories remains conjectural.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Questions about simplicity and rank four

Why is $\text{ch}(G)$ simple exactly when G is simple? Every normal subgroup $N \trianglelefteq G$ gives the fusion subring $\text{ch}(G/N)$ by inflation. Conversely, let R' be a fusion subring, let V be the direct sum of its irreducible basis representations, and put $N = \ker V$. Then V is faithful on G/N .

The matrix coefficients of representations in R' span a unital subalgebra of $\mathbb{C}^{G/N}$: tensor products give closure under multiplication. Faithfulness makes it separate points. A unital point-separating algebra on a finite set is the full function algebra (construct the point indicators by interpolation). The decomposition into irreducible matrix-coefficient spaces implies that R' contains every irreducible representation of G/N . Hence $R' = \text{ch}(G/N)$, proving the equivalence for nontrivial G .

Do not replace $\text{ch}(G)$ by $\mathbb{Z}[G]$. Fusion subrings of $\mathbb{Z}[G]$ correspond to *all* subgroups, not just normal ones; $\mathbb{Z}[G]$ is simple precisely when G is cyclic of prime order. The research question is about based-ring isomorphism, not tensor equivalence to $\text{Rep}(G)$. See (Etingof–Gelaki–Nikshych–Ostrik, AMS, 2015); (Liu–Palcoux–Ren, Internat. J. Math., 2023).

The first Fibonacci example does not answer that question. Its basis is self-dual, with nontrivial products

$$\begin{aligned} X^2 &= 1 + 4Y + 4Z, & XY &= 4X + Y + 6Z, \\ XZ &= 4X + 6Y + 3Z, & Y^2 &= 1 + X + 12Y + Z, \\ YZ &= 6X + Y + 9Z, & Z^2 &= 1 + 3X + 9Y + 6Z. \end{aligned}$$

The positive dimensions are $(1, 11, 14, 16)$ and each nonunit generates the ring. Its formal codegrees are

$$574, \quad 14, \quad 95 + 42\sqrt{5}, \quad 95 - 42\sqrt{5}.$$

The noninteger codegrees exclude categorification unconditionally by the integral result on page 14. The independent calculation $11 \nmid 574$ only demonstrates failure of 1-Frobenius. An algebraic ring construction is not a categorical construction. See (Dong–Palcoux–Plessis, work in progress, 2026); (Aleksyev–Bruns–Dong–Palcoux, arXiv, 2025, Prop. 2.8).

What is asserted about the family? The formula on Board 6 exhibits an infinite family, not an exhaustive parametrization of the scale-one locus or of all simple integral rank-four rings. At $n = 1$ it gives $(1, 14, 16, 11)$ before reordering. The dimension $1 + F_{2n+5}$ increases without bound, and all three nonunit dimensions increase, so the global dimensions increase and the rings are pairwise nonisomorphic. No categorification is asserted. The separate non-1-Frobenius theorem applies to *all* simple integral rank-four rings. See (Dong–Palcoux–Plessis, work in progress, 2026).

Do the near-group categories answer the research question? No. Their d_ρ is irrational, and for $n > 1$ the group basis spans a proper fusion subring. The finale illustrates a different arithmetic connection. See (He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026).

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Arithmetic checks and a second exclusion

Associativity of the two rank-three examples. For the self-dual table

$$X^2 = 1 + mX + kY, \quad XY = kX + \ell Y, \quad Y^2 = 1 + \ell X + nY,$$

associativity is equivalent to $k^2 + \ell^2 = \ell m + kn + 1$. This holds for both $(k, \ell, m, n) = (2, 1, 0, 2)$ and $(3, 2, 3, 2)$. The structure constants are nonnegative integers and the unit/duality conditions hold, so both tables are fusion rings.

Why the discriminant gives an exclusion. For Board 4,

$$\det(tI - N_X) = t^3 - t^2 - 5t + 1, \quad \text{disc}(p) = 592 = 16 \cdot 37.$$

The only possible rational roots are ± 1 , neither of which is a root. For an irreducible cubic, the Galois group is A_3 or S_3 , with the first case equivalent to a square discriminant. Hence it is S_3 . Were the Frobenius–Perron root in a cyclotomic field, the normality of that field would put all three roots there, forcing an abelian splitting field. Equivalently its cubic field is nonnormal and fails the Kronecker–Weber criterion. See ([Washington, Springer, 1997](#), Ch. 14); ([Ostrik, Math. Res. Lett., 2009](#), Ex. 1.6).

Cyclotomicity can pass while the d -number test fails. For the second table,

$$X^2 = 1 + 3X + 3Y, \quad XY = 3X + 2Y, \quad Y^2 = 1 + 2X + 2Y,$$

we have

$$d_X = 3 + \sqrt{7}, \quad d_Y = 2 + \sqrt{7}, \quad D = 1 + d_X^2 + d_Y^2 = 28 + 10\sqrt{7}.$$

The other two ring characters have values $(3 - \sqrt{7}, 2 - \sqrt{7})$ and $(-1, 1)$ on (X, Y) , so all fusion eigenvalues lie in the abelian field $\mathbb{Q}(\sqrt{7})$. But the Frobenius–Perron codegree has

$$p_D(t) = t^2 - 56t + 84, \quad 56^2 \equiv 28 \pmod{84}.$$

Thus $84 \nmid 56^2$, violating the quadratic coefficient criterion. This is a second elementary exclusion, reserved for questions. See ([Ostrik, Math. Res. Lett., 2009](#), Ex. 1.6 and Lem. 2.7).

Codegrees can be checked without finding the character table. Writing N_b for the matrix of multiplication by b , the formal codegrees of a commutative fusion ring are the eigenvalues of $C_R = \sum_{b \in B} N_b N_{b^*}$. For the rank-four example,

$$\det(tI - C_R) = (t - 574)(t - 14)(t^2 - 190t + 205).$$

This gives the four codegrees on page 15. As a further check,

$$\frac{1}{574} + \frac{1}{14} + \frac{190}{205} = 1.$$

Neither the d -number exclusion nor the integral codegree exclusion assumes pivotality or unitarity of a putative complex categorification. See ([Ostrik, Math. Res. Lett., 2009](#)); ([Ostrik, Mosc. Math. J., 2015](#)).

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

Board 7 unpacked: modular data, characters and realization

The conjecture realizes a category, not just matrices. The assertion is $\mathcal{C} \simeq \text{Rep}(V)$ as ribbon tensor categories, with V strongly rational. It requires preservation of the tensor product and associator, braiding and twist. Having the same fusion ring, or even the same modular data, is weaker. The discussion after Conjecture 2.22 in (Möller-Rayhaun, arXiv, 2024, §2.3) records this realization conjecture.

The VOA hypotheses. Strongly rational means simple, self-contragredient, rational, C_2 -cofinite and of CFT type. The last condition is $V = \bigoplus_{m \geq 0} V_m$ with $V_0 = \mathbb{C}\mathbf{1}$. Rationality is a semisimplicity condition on module theory; C_2 -cofiniteness is a finiteness condition. These assumptions give a modular category by (Huang, Commun. Contemp. Math., 2008). For the general realization conjecture, do not add unitarity or strictly positive lowest weights for nonvacuum modules.

What the modular matrices mean. Choose a normalization S of the ribbon Hopf-link matrix; its columns, divided by their unit entry, are the ring characters: $\chi_j(b_i) = S_{ij}/S_{0j}$. The twist on a simple object X_i is the scalar θ_i , represented pictorially by a full framed turn, and $T = \text{diag}(\theta_i)$. The scalars are roots of unity. These matrices supply a projective modular-group representation; its scalar anomaly must be handled when comparing to functions. See (Etingof-Gelaki-Nikshych-Ostrik, AMS, 2015, Ch. 8); (Huang, Proc. Natl. Acad. Sci. USA, 2005).

The character vector and its weight. An irreducible module is $M_i = \bigoplus_{m \geq 0} M_i[h_i + m]$, with finite-dimensional pieces, and

$$F_i(\tau) = q^{h_i - c/24} \sum_{m \geq 0} \dim M_i[h_i + m] q^m, \quad q = e^{2\pi i \tau}.$$

Here h_i is the lowest conformal weight, c the central charge and L_0 the grading operator. The vector $\mathbf{F}$ is holomorphic on the upper half-plane and weakly holomorphic at cusps, with fractional exponents interpreted using the finite-order multiplier. It is a vector-valued modular form of weight 0: for the corresponding linear representation ρ_V ,

$$\mathbf{F}(\gamma\tau) = \rho_V(\gamma)\mathbf{F}(\tau) \quad (\gamma \in \text{SL}_2(\mathbb{Z})),$$

with no weight factor. Individual components need not be scalar modular forms for the full group. See (Zhu, JAMS, 1996); (Huang, Proc. Natl. Acad. Sci. USA, 2005).

Why T and $\widehat{T}$ must be distinguished. The categorical twist is $\theta_i = e^{2\pi i h_i}$, whereas $q^{h_i - c/24}$ transforms by $e^{2\pi i(h_i - c/24)}$. Therefore

$$\rho_V \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = S, \quad \rho_V \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \widehat{T} = e^{-2\pi i c/24} T,$$

where S has the VOA normalization. Modularity of these characters is a theorem; realizing every modular category by such a V is the conjectural step.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

References: fusion categories and obstructions

(Etingof–Gelaki–Nikshych–Ostrik, AMS, 2015)

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SPEAKER BACKUP — OUTSIDE THE PRESENTATION

References: rank four and modular forms

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(Bruillard–Ng–Rowell–Wang, JAMS, 2016)

Rank-finiteness for modular categories. J. Amer. Math. Soc. **29**, 857–881 (2016). [arXiv:1310.7050](#). Theorem 3.9: prime control; Proposition 3.15: the rank-dependent bound on $\text{ord}(T)$; Definition 3.16 and Theorem 3.19: S -units and Evertse. Corollary 3.13 and Theorem 3.1: categorical finiteness. Section 4 uses quantitative bounds.

SPEAKER BACKUP — OUTSIDE THE PRESENTATION

References: unit equations and Stark

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(Evertse, Invent. Math., 1995)

The number of solutions of decomposable form equations. Invent. Math. **122**, 559–601 (1995). Theorem 3 supplies an explicit bound used in Proposition 4.3 of ([Bruillard–Ng–Rowell–Wang, JAMS, 2016](#)); it is not the source selected for the qualitative statement on the board.

(Radchenko–Wheeler, arXiv, Sept. 2026)

Real quadratic fields and finite quantum dilogarithms I. [arXiv:2609.21892](#); first posted 18 September 2026, version 2 dated 29 September 2026. Cyclic near-group existence, analytic invariants and algebraicity through fusion-category rigidity; see the Introduction, Section 2.3, Theorem 8 and Section 5.

(He–Liu–Lu–Palcoux–Ren, arXiv, Oct. 2026)

H. He, Z. Liu, F. Lu, S. Palcoux and Y. Ren, *Triangular prism equations for near-group categories: algebraic solutions and a unitary conjecture.* [arXiv:2610.11652v1](#), posted 8 October 2026 (40 pages). Theorem 2.3: algebraic TPE reconstruction without assuming unitarity, with trivial Frobenius–Schur indicator. Section 2.2 and Corollary 6.6: cyclic construction and pseudo-unitary Galois conjugates. Proposition 6.7: unit-circle criterion. Proposition 6.9: orders $1 \leq n \leq 6$. Conjecture 6.8 and Remark 6.11: the conditional Stark implication and Hilbert’s twelfth problem.

Why the TPE formulation matters (Remark 2.2). The central identity sums degree-two products, whereas Izumi’s corresponding identity sums degree-three products. Radchenko–Wheeler’s construction likewise passes through an identity of the TPE form to solve Izumi’s subsystem. The TPE formulation separates algebraic reconstruction from the additional Hermitian conditions needed for unitarity.

(Appleby–Flammia–Kopp, arXiv, 2025)

A constructive approach to Zauner’s conjecture via the Stark conjectures. [arXiv:2501.03970](#), version 2 (2025). Theorem 2.20(1) gives a positive-power algebraic-unit statement in an abelian extension under Stark; part (2) gives the modulus-one property under root-reversing embeddings.

The two unit connections. In S -unit, S is a set of allowed primes, not an abbreviation for Stark. Board 8 uses a proved finiteness theorem for these units; Board 9 uses conjectural special values and Galois information for particular units. A Stark unit may be an S -unit, but Evertse’s theorem does not assume Stark’s conjecture.