Boundary State Screening Groups in Landau–Ginzburg Models

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Outline

Introduction and main questions

Goal

Define a defect-group-like invariant intrinsically in QFT by classifying boundary charges modulo screening by finite-tension dynamical objects.

- How does the target-space defect group appear in worldsheet QFT?
- What replaces compact versus noncompact branes without a geometric target?
- How do solitons and domain walls screen boundary charges?
- What must happen when screening groups differ across an interface?

Strategy

Extract an integral screening map from BPS data and retain the torsion of its cokernel.

Motivation from noncompact Calabi–Yau spaces

Let X be noncompact, with boundary ∂X:

$H_k(X, \partial X)$ labels noncompact defects, $\qquad$ $H_k(X)$ labels dynamical screening states.

Schematically,

$$D^{(n)} = \mathrm{Tor}\left(\bigoplus_{p-k+1=n} \frac{H_k(X, \partial X)}{H_k(X)} \right).$$

Let Λ_c and $\Lambda_{nc} \simeq \Lambda_c^\vee$ be the compact and noncompact charge lattices. The intersection pairing gives

$$I : \Lambda_c \to \Lambda_{nc}, \qquad I_{ij} = \langle \gamma_i, \gamma_j \rangle, \qquad \boxed{D = \mathrm{Tor}\ \mathrm{coker}(I)}.$$

For a cone, this is equivalently encoded by torsion homology of ∂X.

Outline

$\mathcal{N} = (2,2)$ Landau–Ginzburg models and BPS flows

We work with two-dimensional $\mathcal{N} = (2,2)$ theories of chiral fields Φ^i, specified by a Kähler metric $g_{i\bar{j}}$ and a holomorphic superpotential

$$W : \mathcal{X} \longrightarrow \mathbb{C}.$$

Massive vacua are the isolated critical points a of W: $\partial_i W(a) = 0$. A BPS soliton interpolating from a to b obeys the Morse-flow equation

$$\frac{d\phi^i}{dx} = \frac{e^{i\alpha}}{2} g^{i\bar{j}} \partial_{\bar{j}} \overline{W}, \qquad \alpha = \arg(W_b - W_a).$$

Its tension satisfies

$$T_{ab} \geq |W_b - W_a|,$$

with equality for a BPS flow. Signed counts of these finite-tension trajectories provide the integral screening data.

Boundary charges and soliton screening

Generalized boundary states are graded by a charge lattice:

$$\mathcal{H}_{\rm bdry} = \bigoplus_{\lambda\in\Lambda} \mathcal{H}^{(\lambda)}_{\rm bdry}.$$

A finite-tension soliton carries $[\sigma] \in \Lambda_{\rm sol}$ and shifts a boundary charge upon absorption:

$$s : \Lambda_{\rm sol} \to \Lambda, \qquad \lambda \mapsto \lambda + s([\sigma]), \qquad \Lambda_{\rm scr} = \mathrm{im}(s).$$

Therefore

$$\mathcal{B}_{\mathcal{T}} = \Lambda/\Lambda_{\rm scr} = \mathrm{coker}(s), \qquad \boxed{B_{\mathcal{T}} = \mathrm{Tor}\ \mathrm{coker}(s)}.$$

The free part records asymptotic charge data; the torsion part is finite, discrete, and unscreened.

Matrix presentation and LG realization

Putting the screening charges $q_\alpha = s([\sigma_\alpha])$ into columns gives an integral matrix Q_{sol}:

$$\mathcal{B}_{\mathcal{T}} = \mathrm{coker}(Q_{\mathrm{sol}}).$$

Its Smith normal form determines the finite group:

$$UQ_{\mathrm{sol}}V = \mathrm{diag}(d_1, \ldots, d_r, 0, \ldots, 0), \qquad B_{\mathcal{T}} = \bigoplus_{d_i > 1} \mathbb{Z}_{d_i}.$$

For a massive LG model, compact vanishing cycles generate Λ_{sol}, while noncompact Lefschetz thimbles generate the dual lattice Λ. The compact pairing is the screening map

$$K : \Lambda_{\mathrm{sol}} \to \Lambda_{\mathrm{sol}}^\vee \simeq \Lambda, \qquad \boxed{B = \mathrm{Tor}\,\mathrm{coker}(K)}.$$

From BPS data to the screening matrix

Let μ be the strictly upper-triangular matrix of signed BPS soliton counts, and set

$$S = 1 + \mu.$$

For an LG model with n chiral fields,

$$\boxed{K = S + (-1)^{n-1} S^T}.$$

Consequences:

- n even: $K = S - S^T$ is antisymmetric;
- n odd: $K = S + S^T$ is symmetric.

For an orbifold LG phase, n is inherited from its parent GLSM rather than counted naively after orbifolding.

Outline

Orbifold mirrors: cycles, solitons, and quivers

For a two-field mirror $W(x,y)$, vacua satisfy $\partial_x W = \partial_y W = 0$. For each vacuum $r \in \mathbb{Z}_N$ there are dual cycles

$$\Gamma_r \text{ (noncompact thimble)}, \qquad C_r \text{ (compact cycle)}, \qquad \langle \Gamma_r, C_s \rangle = \delta_{rs}.$$

Thus

$$\Lambda = \bigoplus_r \mathbb{Z}[\Gamma_r], \qquad \Lambda_{\mathrm{sol}} = \bigoplus_r \mathbb{Z}[C_r], \qquad K = S - S^T.$$

The orbifold quiver gives the same matrix. If

$$A_{rs} = \sum_{\ell=1}^{3} \delta^{(N)}_{s,r+a_\ell},$$

then

$$K = A^T - A, \qquad B = \mathrm{Tor}\,\mathrm{coker}(A^T - A).$$

$\mathbb{C}^3/\mathbb{Z}_3(1,1,1)$

The effective LG superpotential is

$$W(x,y) = x + y + \frac{1}{xy}.$$

Critical points satisfy

$$x = y, \qquad x^3 = 1.$$

Thus

$$v_r = (\omega^r, \omega^r), \qquad W_r = 3\omega^r, \qquad r = 0, 1, 2.$$

The critical values form an equilateral triangle in the W-plane.

Stokes and screening matrices

A distinguished basis gives

$$S = \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}.$$

Therefore

$$K = S - S^T = \begin{pmatrix} 0 & -3 & 3 \\ 3 & 0 & -3 \\ -3 & 3 & 0 \end{pmatrix}.$$

The factor of three remembers the BPS multiplicity invisible in the unweighted soliton graph.

Smith normal form and the answer

The Smith normal form is

$$\mathrm{SNF}(K) = \mathrm{diag}(3, 3, 0).$$

Hence

$$\mathcal{B} = \mathrm{coker}(K) \simeq \mathbb{Z} \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3,$$

and

$$\boxed{B \simeq \mathbb{Z}_3 \oplus \mathbb{Z}_3}.$$

Equivalently,

$$\mathrm{im}(K) = 3Q(A_2), \qquad B \simeq Q(A_2)/3Q(A_2).$$

A non-geometric test: A-type minimal models

The level-k model arises from $W_0(X) = X^{k+2}/(k+2)$. Resolve its degenerate critical point by

$$W(X) = \frac{1}{k+2} X^{k+2} - X, \qquad W'(X) = X^{k+1} - 1.$$

There are $k+1$ massive vacua and critical values

$$X_a = e^{2\pi i a/(k+1)}, \qquad w_a = -\frac{k+1}{k+2} X_a, \qquad a = 0, \dots, k.$$

There is one BPS soliton between each pair of distinct vacua. With a suitable ordering,

$$\mu_{ab} = \begin{cases} 1, & a < b, \\ 0, & a \geq b, \end{cases}$$

so $S = 1 + \mu$ has ones on and above the diagonal.

Screening group of the A_k model

There is one LG field, so

$$K = S + S^T.$$

Writing $m = k + 1$,

$$K = I_m + J_m,$$

where J_m is the all-ones matrix. Its eigenvalues are 1 with multiplicity $m - 1$ and $m + 1 = k + 2$ once. Moreover,

$$\mathrm{SNF}(K) = \mathrm{diag}(1, \ldots, 1, k + 2).$$

Therefore

$$\boxed{B \simeq \mathbb{Z}_{k+2}}.$$

Compact nongeometric test: the 1^9 model

Consider the diagonal orbifold

$$\widetilde{W} = \sum_{i=1}^{9} y_i^3, \qquad G = \langle J \rangle \simeq \mathbb{Z}_3.$$

Rather than imposing a sign convention, construct the integral orbifold A-brane lattice directly. With $S = S_{A_2}^{\otimes 9}$, $M = C_{A_2}^{\otimes 9}$, and $N = 1 + M + M^2$,

$$\Lambda_{\mathrm{orb}} = \mathrm{im}_{\mathbb{Z}}\, N = \ker_{\mathbb{Z}}(M-1), \qquad \mathrm{rk}\, \Lambda_{\mathrm{orb}} = \frac{512 - 1 - 1}{3} = 170.$$

For an integral basis B, exact Hermite reduction gives a primitive lattice. The normalized pairing is automatically antisymmetric and obeys

$$K_{\mathrm{orb}} = \frac{1}{3} B^T S B, \qquad \mathrm{SNF}(K_{\mathrm{orb}}) = \mathrm{diag}(1^{170}).$$

This alternative derivation and computation were AI generated.

Compactness test passed

$\boxed{\mathcal{B}_{1^9} = \mathrm{Tor}\,\mathrm{coker}\, K_{\mathrm{orb}} = 0.}$ Twisted/fractional A-branes complete the cover to a unimodular lattice.

Outline

Application: A- and D-type Argyres–Douglas theories

The relevant non-orbifold $\mathcal{N} = (2,2)$ LG models are defined directly by

$$W_A = x^2 + y^2 + z^2 + w^k, \qquad W_D = x^2 + y^2 + z^{k-1} + zw^2.$$

The same quasihomogeneous polynomials define the corresponding noncompact Calabi–Yau threefold singularities through $W_A = 0$ and $W_D = 0$. No parent GLSM or additional Lagrange-multiplier field is involved.

The sign convention applies directly

Each superpotential has $n = 4$ chiral fields. Therefore

$$K_{\mathrm{AD}} = S + (-1)^{n-1} S^T = S - S^T, \qquad D_{\mathrm{AD}} = \mathrm{Tor\ coker}\, K_{\mathrm{AD}}.$$

The A/D soliton numbers determine S, and the resulting antisymmetric pairing agrees with the intersection pairing of special Lagrangian three-cycles.

Screening groups of A- and D-type AD theories

The antisymmetrized Stokes matrix is integrally equivalent to an oriented Dynkin incidence matrix. For A_{k-1},

$$\mathrm{SNF}(K_A) = \begin{cases} \mathrm{diag}(1^{k-1}), & k \text{ odd}, \\ \mathrm{diag}(1^{k-2}, 0), & k \text{ even}. \end{cases}$$

For D_k,

$$\mathrm{SNF}(K_D) = \begin{cases} \mathrm{diag}(1^{k-2}, 0, 0), & k \text{ even}, \\ \mathrm{diag}(1^{k-1}, 0), & k \text{ odd}. \end{cases}$$

Hence the cokernel contains only free flavor-charge directions:

$$\mathcal{B}_{A_{k-1}} = \begin{cases} 0, & k \text{ odd}, \\ \mathbb{Z}, & k \text{ even}, \end{cases} \qquad \mathcal{B}_{D_k} = \begin{cases} \mathbb{Z}^2, & k \text{ even}, \\ \mathbb{Z}, & k \text{ odd}, \end{cases} \qquad \boxed{B_{A_{k-1}} = B_{D_k} = 0}.$$

Thus these AD theories have no finite unscreened defect charge in this construction.

Outline

Adding an interface

An interface is a codimension-one object separating

$$\mathcal{T}_1 \mid \mathcal{T}_2.$$

It may be engineered by allowing masses, couplings, or superpotential parameters to vary across space.

The interface is itself a QFT

Its $(D-1)$-dimensional worldvolume can carry localized matter, gauge dynamics, and its own defect sectors.

A bulk defect reaching the wall must either transmit or terminate there.

Folding: interfaces as boundary conditions

Reflect the right-hand theory across the wall:

$$\widetilde{\phi}_2(x,t) = \phi_2(-x,t), \qquad x \le 0.$$

Then an interface becomes a boundary condition for

$$\mathcal{T}_1 \otimes \overline{\mathcal{T}_2}.$$

This lets us apply the boundary-screening viewpoint to interfaces:

- bulk charges can be identified across the wall;
- unmatched charges can end on the folded brane;
- localized interface excitations provide additional screening.

Supersymmetric LG interfaces

For an A-type interface between W_1 and W_2,

$$(W_1 - \overline{W}_1)\big|_{-0} = (W_2 - \overline{W}_2)\big|_{+0}\,.$$

After folding, the combined superpotential is

$$W_{\rm fold}(\phi, \chi) = W_1(\phi) - W_2(\chi).$$

The folded interface is an A-brane satisfying

$$(e^{i\alpha} W_{\rm fold} - e^{-i\alpha}\overline{W}_{\rm fold})\big|_{\gamma} = \text{constant}.$$

Reflective and transmissive walls

The behavior of $\operatorname{Im} W$ distinguishes two basic cases:

$$\begin{aligned} \partial_x \operatorname{Im} W_1 = \partial_x \operatorname{Im} W_2 = 0 \quad &\Longrightarrow \quad \text{reflective}, \\ \partial_x \operatorname{Im} W_1 = \partial_x \operatorname{Im} W_2 \neq 0 \quad &\Longrightarrow \quad \text{transmissive}. \end{aligned}$$

For a transmissive interface, a soliton on the left can continue as a soliton on the right. The interface glues their BPS trajectories while preserving the appropriate supercharge.

Refraction in the W-plane

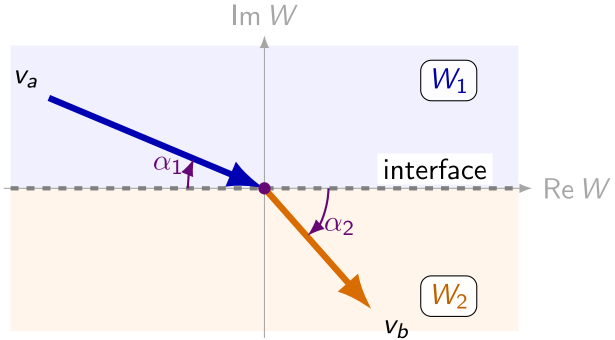
An intrinsically $\mathcal{N} = (1,1)$ interface may obey

$$\partial_x W|_{-0} = \partial_x W|_{+0} + \delta e^{i\sigma}.$$

The jump changes the BPS phase,

$$\alpha_1 \longrightarrow \alpha_2, \qquad \alpha_1 \neq \alpha_2,$$

so the trajectory refracts and the wall carries tension.



Screening groups across an interface

Let B_1 and B_2 be the bulk screening groups. An interface must specify which charges

- transmit from B_1 to B_2,
- terminate on the wall,
- are screened by localized interface degrees of freedom.

Mismatch criterion

If a bulk charge has no transmitted image, it cannot disappear: the interface must act as its source or sink.

A mismatch in screening groups or anomalies therefore obstructs a trivially gapped interface.

Tuning refraction changes interface screening

For folded vacua (a, α), the critical values are

$$Z_{a\alpha} = W_1(v_a) - W_2(v_\alpha).$$

Changing (δ, σ) can create refracted flows $(a, \alpha) \to (b, \beta)$ with charge

$$q_{a\alpha, b\beta} = e_{b\beta} - e_{a\alpha}.$$

These flows add screening relations:

$$B_{\mathcal{I}} = \mathrm{Tor}\, \frac{\Gamma^\vee}{K\Gamma + \langle q_1, \ldots, q_N \rangle}, \qquad B_{\mathrm{fold}} \twoheadrightarrow B_{\mathcal{I}}.$$

Chamber dependence

$B_{\mathcal{I}}$ is constant under generic parameter changes and jumps only when a refracted trajectory appears or disappears.

If $B_{\mathrm{fold}} = (\mathbb{Z}_2)^t$, independent refracted solitons can partially or completely screen it; a maximal isotropic set leaves $H^\perp / H = 0$.

Outline

Summary

1. Boundary charges are screened by absorption of finite-tension objects.
2. The intrinsic invariant is
$$B_{\mathcal{T}} = \mathrm{Tor}\ \mathrm{coker}(s).$$
3. In massive LG models, s is the compact pairing
$$K = S + (-1)^{n-1} S^T.$$
4. Examples:
$$\mathbb{C}^3/\mathbb{Z}_3 : \quad \mathbb{Z}_3 \oplus \mathbb{Z}_3, \qquad A_k : \quad \mathbb{Z}_{k+2}.$$
5. The construction lifts from solitons to domain walls and constrains interfaces.

Outlook

- Precise functorial formulation for interfaces and junctions.
- Wall crossing and invariance of the Smith data.
- Relation to generalized global symmetries and higher-form charges.
- Extension to categories of branes beyond charge lattices.
- Applications to non-geometric phases and strongly coupled QFTs.

Thank you