Stable degenerations of Calabi-Yau manifolds and their role in Quantum Gravity

- 2504.01066, 2509.07056 and 2510.0243 with (**Björn Hassfeld)**, **Jeroen Monnee** and **Max Wiesner**
- 2609.XXXX with **Lukas Kaufmann**
- 2603.12315 and 2603.13470 with **Lukas Kaufmann, Jeroen Monnee** and **Max Wiesner**

Timo Weigand, 50th Anniversary of Calabi-Yau, Tsinghua University, Beijing, September 7, 2026

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DFG German-Israeli Project Cooperation (DIP) Holography and the Swampland

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50 years of Yau's theorem

Reason to celebrate decade-long

cross-fertilization of

algebraic geometry and fundamental physics

50 years of Yau's theorem

Reason to celebrate a miracle:

The (un)reasonable effectiveness of

Calabi-Yau geometry in Quantum Gravity

Motivation

In Quantum Gravity, all physical couplings are (believed to be) dynamical

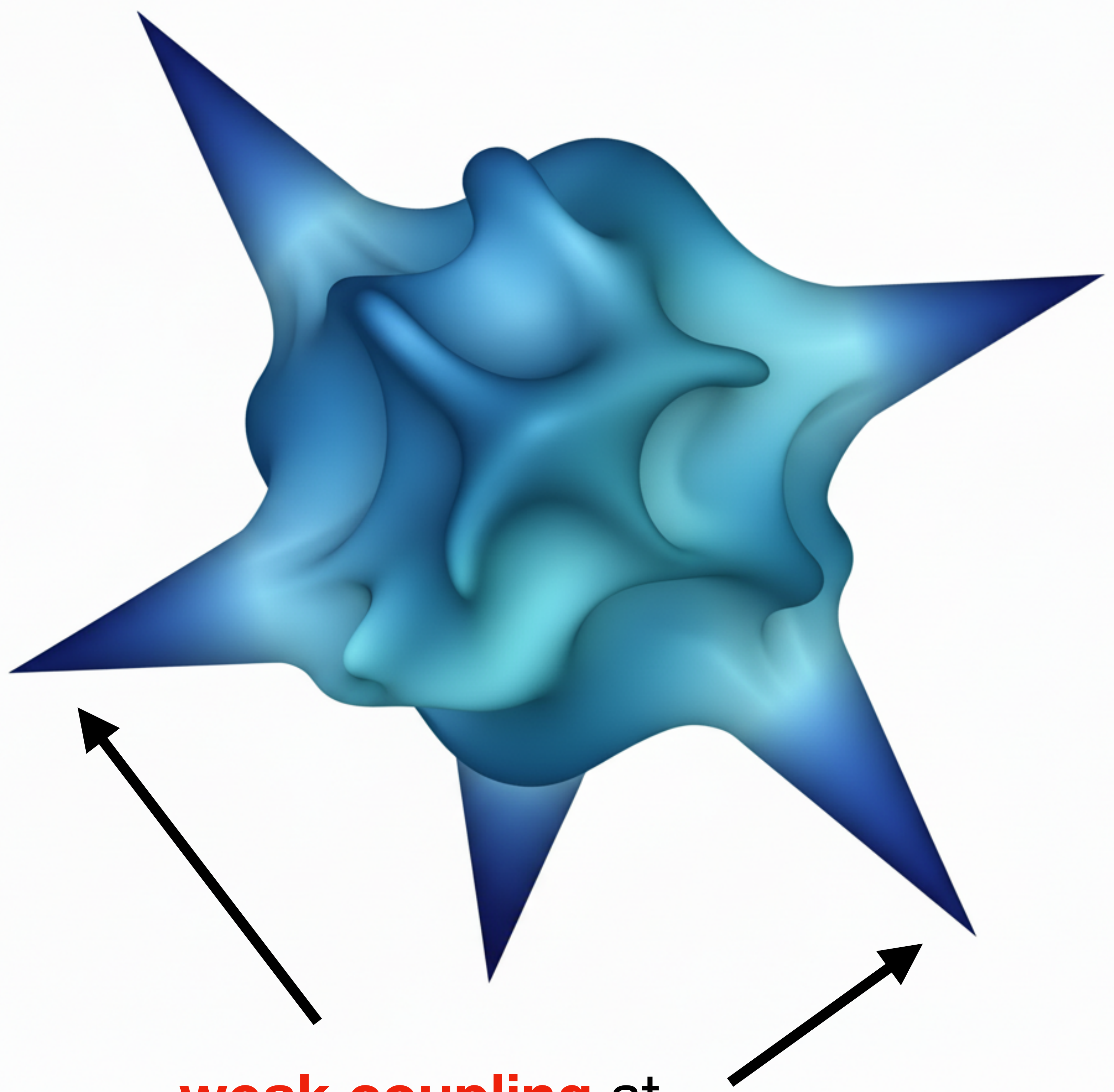
and functions on a (pseudo-)moduli space $\mathscr{M}$.

$$S = \int \frac{1}{2g^2(\varphi)} F^2 + \ldots + G_{ij}(\varphi)\, \partial_\mu \varphi^i\, \partial^\mu \varphi^j$$

coupling

dynamical scalar fields

metric on the moduli space



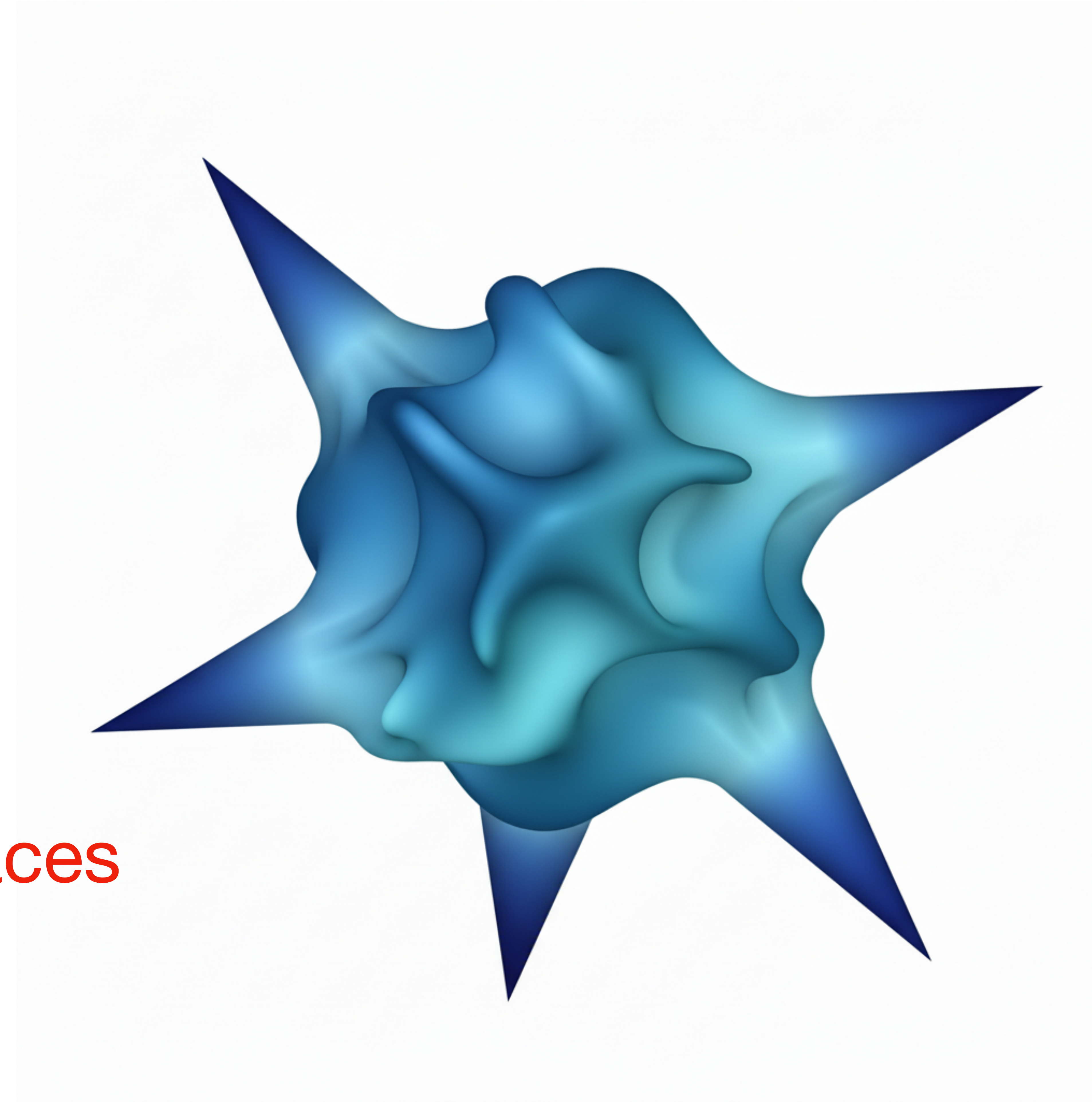
weak coupling at infinite distance in moduli space:

$\Delta \to \infty$

Motivation

In Quantum Gravity, all physical couplings are (believed to be) dynamical

and functions on a (pseudo-)moduli space $\mathscr{M}$.



Which properties must the (pseudo-)moduli spaces of consistent QG theories have?

Motivation

Distance Conjecture

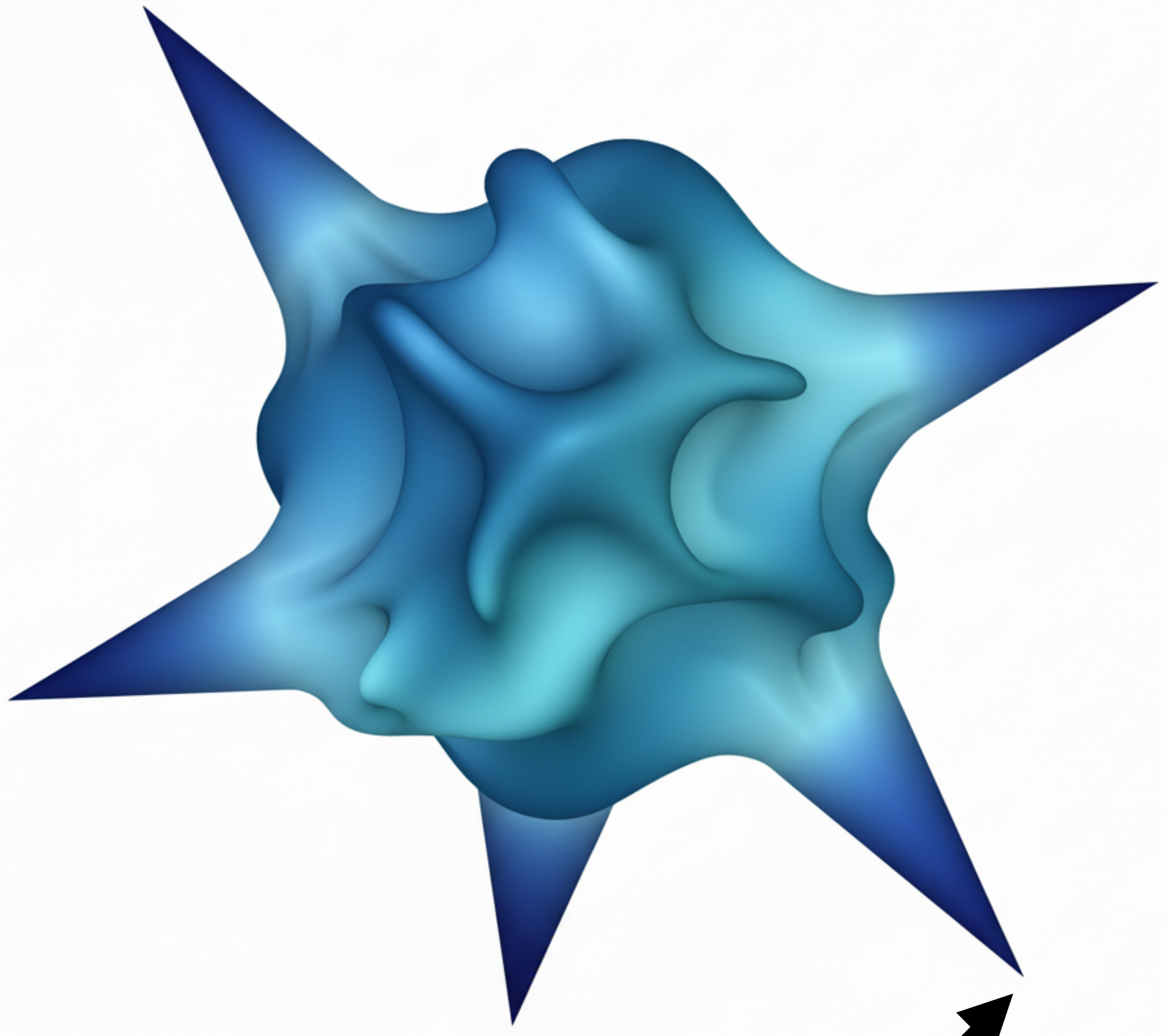
[Ooguri,Vafa'06]

Tower of gravitational states becomes lights

as

distance on moduli space $\mathscr{M}$

$m \sim m_0\, e^{-\alpha\Delta}$ for $\Delta \to \infty$



infinite distance $\Delta \to \infty$

Emergent String Conjecture

[Lee,Lerche,TW'19]

All such towers have a dual description as

- KK towers
- or fundamental string excitations.

General arguments: [Hamada,Montero,Valenzuela,Vafa'21] [Draper,Farkas'19] [Bonnefoy,Ciambelli,D. Lüst,S. Lüst'19] [Calderon-Infante,Castellano,Herraez,Ibanez'23]

General arguments: [Cribiori, Lüst, Montella'23] [Basile, Lüst, Montella'23] [Basile,Cribiori,Lüst, Montella'24] [Herraez,Lüst,Masias,Scalisi'24] [Bedroya,Mishra,Wiesner'24]

Motivation

- How constraining are these (and other) criteria on moduli spaces?
  Which classes of theories are ruled out by them?

- Which spaces can be moduli spaces of consistent QG theories?
  Which spaces can be moduli spaces of Calabi-Yau manifolds?

- Conversely:

  Given a consistent QG - such as string theory on a Calabi-Yau manifold - **which properties of the CY make the QG criteria to hold?**

  Can we learn something from physics for the CY geometry?

Motivation

Emergent String Conjecture

[Lee,Lerche,TW'19]

1) Constrains tower mass decay rate $m \sim m_0\, e^{-\alpha \Delta}$

$$\alpha_{\rm KK} = \sqrt{\frac{1}{d-2} + \frac{1}{n}}$$

decompactification
$d \to d+n$

$$\alpha_s = \frac{1}{\sqrt{d-2}}$$

[Etheredge, Heidenreich, Kaya,Qiu,Rudelius '22]
[Agmon,Bedroya,Kang,Vafa'22]

2) Constrains tower spectrum and degeneracy

$$\text{KK}: \quad m_n = n M_t\,, \qquad d_n \sim n^{p-1}$$

$$\text{string}: \quad m_n = \sqrt{n} M_t\,, \qquad d_n \sim e^{c\sqrt{n}}$$

Studied in detail especially in **vector multiplet moduli spaces of string compactifications on CYs**

Kähler moduli: [Lee,Lerche,TW'18 '19' 20] [Corvilain,Grimm,Valenzuela'18] [Xu'20] [Lee,Kläwer,TW,Wiesner'20] [Baume,Marchesano,Wiesner'20][Blumenhagen,Gligovic,Paraskevopoulo'23] [Kaufmann,TW'24], ….

Main message of this talk

For vector multiplets in theories with 8 supercharges:

1) Constrains tower mass decay rate $m \sim m_0\, e^{-\alpha\Delta}$

$$\alpha_{\rm KK} = \sqrt{\frac{1}{d-2} + \frac{1}{n}}$$

decompactification
$d \to d + n$

$$\alpha_s = \frac{1}{\sqrt{d-2}}$$

[Etheredge, Heidenreich, Kaya,Qiu,Rudelius '22]
[Agmon,Bedroya,Kang,Vafa'22]

Scalings guaranteed in any theory with eight real supercharges by properties of supersymmetry alone!

[Kaufmann,TW, to appear]

2) Constrains tower spectrum and degeneracy

$$\mathrm{KK}: \quad m_n = nM_t\,, \qquad d_n \sim n^{p-1}$$

$$\mathrm{string}: \quad m_n = \sqrt{n}M_t\,, \qquad d_n \sim e^{c\sqrt{n}}$$

Requires detailed input from QG / CY geometry:
Stable degenerations of CY 3-folds

[Lee,Lerche,TW'19]
[(Hassfeld),Monnee,TW,Wiesner'25]

Part I:

Analysis in vector multiplet sector of 4d N=2 supergravity

4d N=2 supergravity

$$S_{4d} = \int_{\mathbb{R}^{1,3}} \left(\frac{M_{\text{Pl}}^2}{2} \star R - 2g(t,\bar{t})_{i\bar{j}} \mathrm{d}t^i \wedge \star \mathrm{d}\bar{t}^{\bar{j}} + \frac{1}{2}\mathrm{Im}\mathcal{N}_{IJ}(t,\bar{t})\mathcal{F}^I \wedge \star \mathcal{F}^J + \frac{1}{2}\mathrm{Re}\mathcal{N}_{IJ}(t,\bar{t})\mathcal{F}^I \wedge \mathcal{F}^J \right)$$

$I : 0, \quad 1, \ldots, n$

$t^i = \frac{Z^i}{Z^0}$

t^i : **complex scalar fields (vector multiplet moduli)**

graviphoton

U(1) gauge fields $A^i \quad i = 1, \ldots, n$

Special Kähler manifold $\mathcal{M}_n$ with coordinates t^i

1) $\boxed{g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} K}$ $\qquad e^{-K} = 2\mathrm{Im}\left(Z^I \bar{F}_I\right)$ $\qquad$ **prepotential** $F = F(Z^I)$ $\qquad F_I = \partial_I F\,, \qquad F_{IJ} = \partial_I \partial_J F$

2) $\boxed{\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2i\frac{\mathrm{Im}F_{IK}Z^L\,\mathrm{Im}F_{JL}Z^L}{Z^M \mathrm{Im}F_{MN}Z^N}}$

4d N=2 supergravity

Period vector $\Pi = \begin{pmatrix} Z^I \\ F_I \end{pmatrix}$

section of vector bundle H over M_n of rank $2(n+1)$

$\longrightarrow$ symplectic pairing $\langle\, . \,, . \,\rangle$ $\quad e^{-K} = 2\mathrm{Im}(Z^I \bar{F}_I) = i\langle \bar{\Pi}, \Pi \rangle$

$\longrightarrow$ $H_{\mathbb{C},t}$: complexification of fiber of H over $t \in M_n$ $\quad V = e^{K(t,\bar{t})/2}\Pi(t)$

Hodge structure of weight 3:

$$H_{\mathbb{C},t} = \bigoplus_{p+q=3} H_t^{p,q}\,, \qquad \overline{H_t^{p,q}} = H_t^{q,p}$$

[Ferrara,Louis'91]
[Ceresole,D'Auria,Ferrara,Lerche,Louis'92]

$$H_t^{3,0} := \mathrm{span}_{\mathbb{C}}\{V\}$$
$$H_t^{2,1} := \mathrm{span}_{\mathbb{C}}\{D_i V\} \qquad D_i = \partial_i + pK,_i$$
$$H_t^{1,2} := \overline{H_t^{2,1}} = \mathrm{span}_{\mathbb{C}}\{\overline{D_i V}\}$$
$$H_t^{0,3} := \overline{H_t^{3,0}}$$

4d N=2 supergravity

Hodge structure of weight 3:

$$H_{\mathbb{C},t} = \bigoplus_{p+q=3} H_t^{p,q}\,, \qquad \overline{H_t^{p,q}} = H_t^{q,p}$$

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$$H_t^{1,2} := \overline{H_t^{2,1}} = \mathrm{span}_{\mathbb{C}}\{\overline{D_i V}\}$$
$$H_t^{0,3} := \overline{H_t^{3,0}}$$

$$V = e^{K(t,\bar{t})/2}\Pi(t)$$

Polarisation from symplectic pairing:

$$(H_t^{p,q}, H_t^{r,s}) = 0 \qquad \text{unless } (p,q) = (r,s)$$
$$i^{p-q}(v,\bar{v}) > 0 \qquad \text{for } v \in H_t^{p,q}\backslash\{0\}$$

Hodge norm:

$$\|v\|^2 = (v, C\bar{v})\,, \quad v \in H_{\mathbb{C},t}$$

$$C = \begin{pmatrix} 0 & -\mathbb{I}_{n+1} \\ \mathbb{I}_{n+1} & 0 \end{pmatrix} \mathcal{M}$$

→ Gauge couplings for $q^I A_I$

$$\mathfrak{q}_q^2 = -\frac{1}{2} q^T \mathcal{M} q = \frac{1}{2}(q, Cq) = \frac{1}{2}\|q\|^2$$

$$\mathcal{M} = \begin{pmatrix} \mathrm{Im}(\mathcal{N}) + \mathrm{Re}(\mathcal{N})\big(\mathrm{Im}(\mathcal{N})\big)^{-1}\mathrm{Re}(\mathcal{N}) & -\mathrm{Re}(\mathcal{N})\big(\mathrm{Im}(\mathcal{N})\big)^{-1} \\ -\big(\mathrm{Im}(\mathcal{N})\big)^{-1}\mathrm{Re}(\mathcal{N}) & \big(\mathrm{Im}(\mathcal{N})\big)^{-1} \end{pmatrix}$$

→ Kähler potential:

$$e^{-K} = \|\Pi\|^2$$

4d N=2 supergravity

Hodge structure of weight 3:

$$H_{\mathbb{C},t} = \bigoplus_{p+q=3} H_t^{p,q}\,, \qquad \overline{H_t^{p,q}} = H_t^{q,p}$$

$$\begin{aligned} H_t^{3,0} &:= \mathrm{span}_{\mathbb{C}}\{V\} \\ H_t^{2,1} &:= \mathrm{span}_{\mathbb{C}}\{D_i V\} \\ H_t^{1,2} &:= \overline{H_t^{2,1}} = \mathrm{span}_{\mathbb{C}}\{\overline{D_i V}\} \\ H_t^{0,3} &:= \overline{H_t^{3,0}} \end{aligned}$$

4d N=2 supersymmetry alone: **[Ferrara,Louis'91]** **[Ceresole,D'Auria,Ferrara,Lerche,Louis'92]**

$$V = e^{K(t,\bar{t})/2}\Pi(t)$$

$\Longrightarrow$ **Variation of polarized (complex) Hodge structure (VPHS)**

Example: Griffiths transversality:

$$F_t^p = \bigoplus_{q\geq p} H_t^{q,3-q}\,, \qquad \frac{\partial F^p}{\partial t^i} \subseteq F^{p-1}\,, \quad \frac{\partial F^p}{\partial \bar{t}^i} \subseteq F^p$$

$$\begin{aligned} D_i\Pi &= U_i \\ D_i U_j &= -\,iW_{ijk}g^{k\bar{k}}\bar{U}_{\bar{k}} \\ D_i\bar{U}_{\bar{j}} &= g_{i\bar{j}}\bar{\Pi} \\ D_i\bar{\Pi} &= 0 \end{aligned}$$

Polarized Hodge Structures in N=2 SUGRA

4d N=2 supersymmetry/supergravity alone:

- Variation of polarized ***complex*** Hodge structure (VPHS)
- A priori ***no underlying integral*** structure in supergravity without further assumptions

For VPHS with ***integral structures:*** $H_{\mathbb{C},t} = H_{\mathbb{Z},t} \otimes \mathbb{C}$

- Strong results determine e.g. asymptotic scaling of Hodge norm

[Schmid'73]
[Cattani,Caplan,Schmid'86]

- Applied to complex structure limits in Type IIB string theory on CY 3-folds

[Grimm,Palti,Valenzuela'18] [Grimm,Li,Palti'18] …. Reviews + refs: [v.d. Heisteeg'22] [Monnee'24]

For complex VPHS ***(no integral structure)***: recent results by **[Sabbah,Schnell'22 + to appear]**
cf. **[Deng'22]**

Limiting Mixed Hodge Structure in N=2 SUGRA

Single **parameter infinite distance limit:**

$$\frac{1}{2\pi i}\log z = t = a + is \to i\infty$$

[Cattani,Kaplan,Schmid'86] …
[Sabbah,Schnell'22]

$$H^3(V,\mathbb{C}) = \bigoplus_{0\le p,q\le 3} I^{p,q} = \bigoplus_{\ell_r=0}^{6} \mathrm{Gr}_{\ell_r}$$

$$\|q\|^2 \sim s^{\ell-3} \qquad q \in \mathrm{Gr}_\ell$$

Limiting Mixed Hodge Structure in N=2 SUGRA

Single **parameter infinite distance limit:**

$$\frac{1}{2\pi i}\log z = t = a + \mathrm{i}s \to \mathrm{i}\infty$$

$$\|q\|^2 \sim s^{\ell-3} \qquad q \in \mathrm{Gr}_\ell$$

[Cattani,Kaplan,Schmid'86] ...

[Sabbah,Schnell'22]

1) $e^{-K} = \|\Pi\|^2 \sim s^d$ since $\Pi \in \mathrm{Gr}_{3+d}$ since $d = 1,2,3$ for type II, III, IV

In CY context:

[Grimm,Palti,Valenzuela'18]

[Grimm,Li,Palti'18] ....

[v.d. Heisteeg'22] [Monnee'24]

$$\Longrightarrow \qquad \triangle = \int_1^\lambda \mathrm{d}s\sqrt{\frac{1}{2}\frac{\mathrm{d}^2K}{\mathrm{d}(s)^2}} = \sqrt{\frac{d}{2}}\log\lambda$$

distance in moduli space

2) $$\mathfrak{q}_{\min} \sim (s)^{(\ell-3)/2} = \exp\left(-\sqrt{\frac{d}{2}}\,\triangle\right) \equiv \exp\left(-\alpha_{\mathrm{A}}\,\triangle\right)$$

$$\alpha_{\mathrm{II}} = 1/\sqrt{2}, \quad \alpha_{\mathrm{III}} = 1, \quad \alpha_{\mathrm{IV}} = \sqrt{3/2}$$

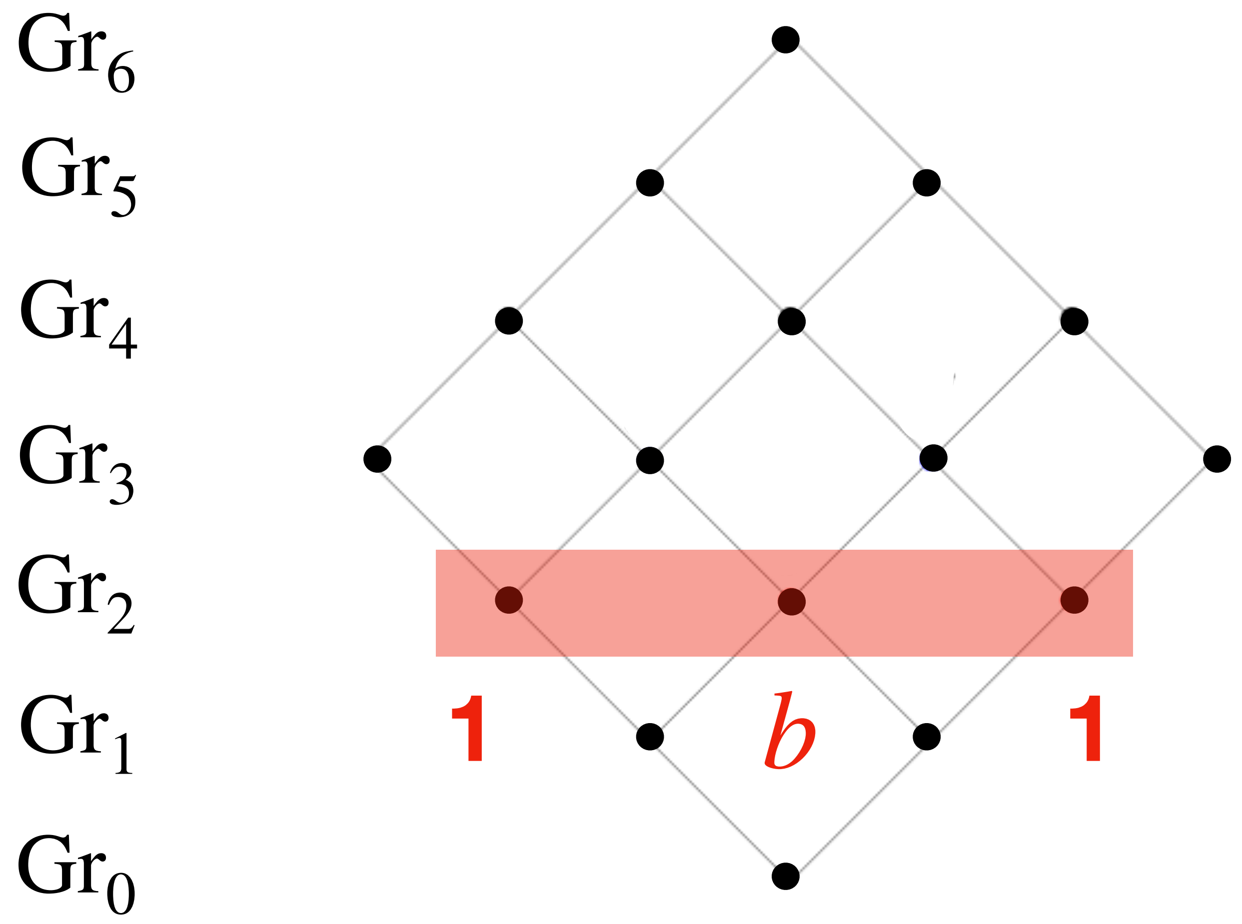
Limiting Mixed Hodge Structure in N=2 SUGRA

Single parameter infinite distance limit:

$$\frac{1}{2\pi i}\log z = t = a + \mathrm{i}s \to \mathrm{i}\infty$$

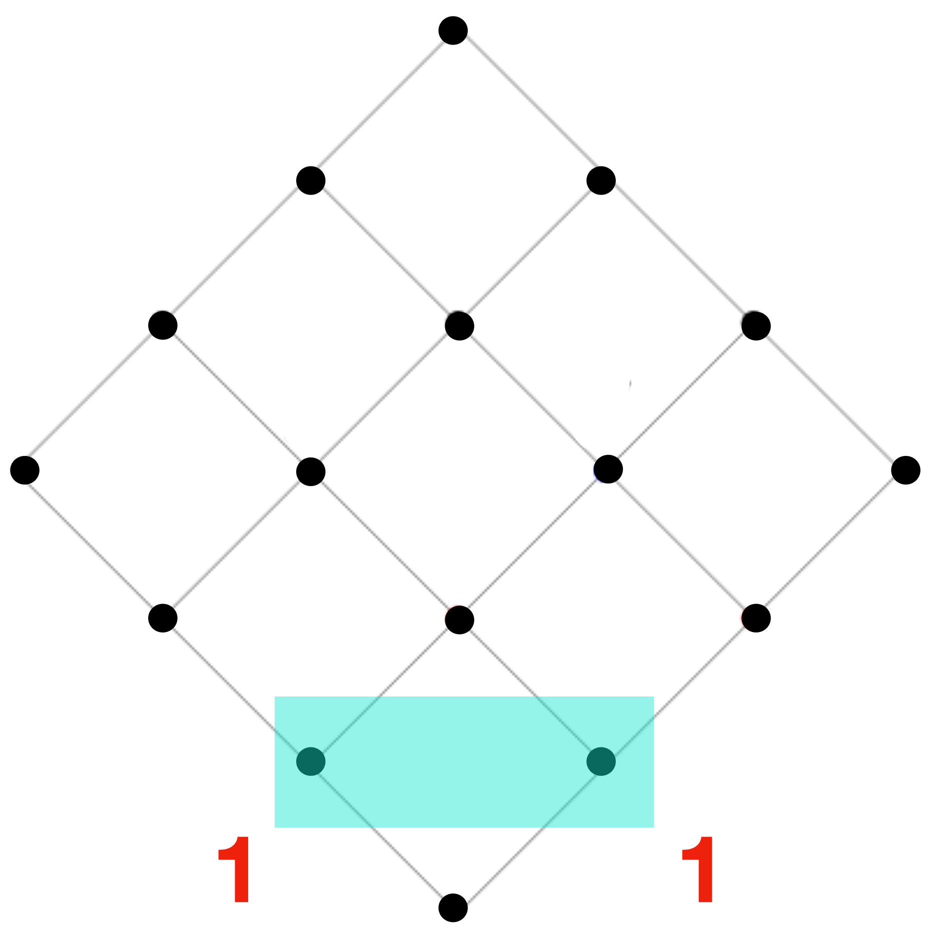
[Cattani,Kaplan,Schmid'86] ...
[Sabbah,Schnell'22]

type II$_a$



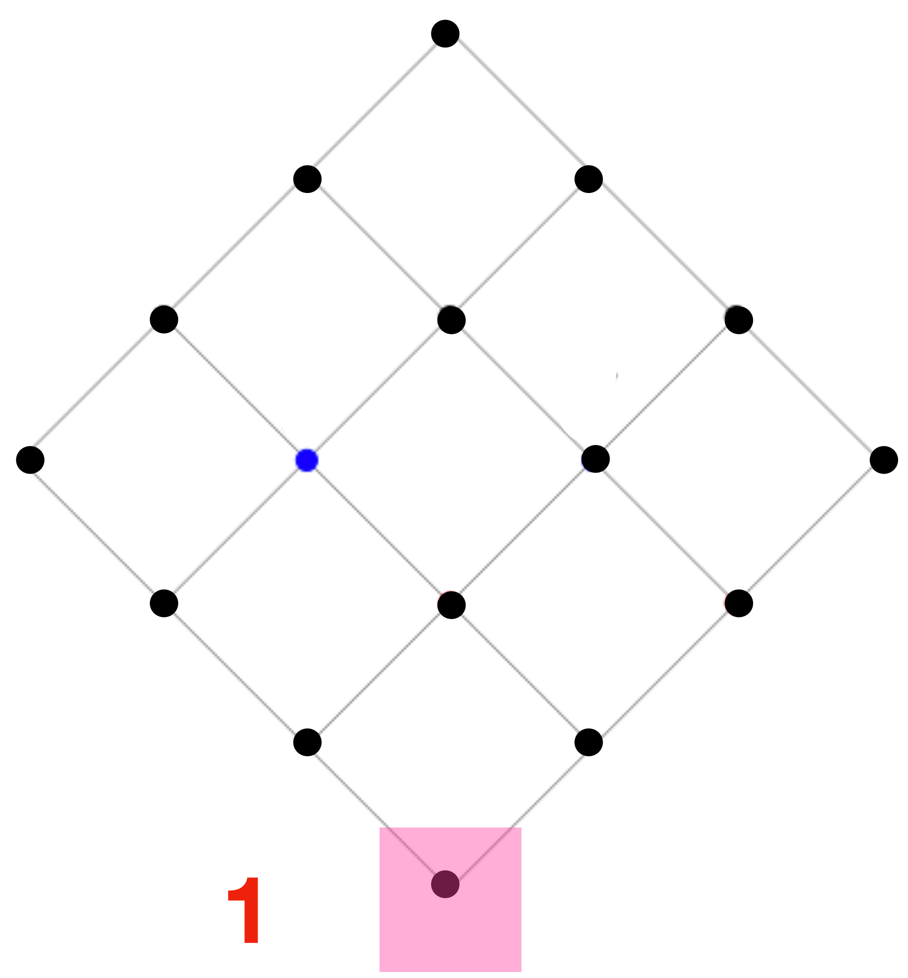
$$\mathfrak{q}^a_{\min} \sim \exp\left(-\sqrt{\frac{1}{2}}\,\triangle\right)$$

type III$_c$



$$\mathfrak{q}^i_{\min} \sim \exp\left(-\,\triangle\right)$$

type IV$_d$



$$\mathfrak{q}_{\min} \sim \exp\left(-\sqrt{\frac{3}{2}}\,\triangle\right)$$

Consistent with:

Weakly coupled string limit

Decompactification $4d \to 6d$

Decompactification $4d \to 5d$

Limiting Mixed Hodge Structure in N=2 SUGRA

Generalisation to higher-dimensional limits under **condition of simple normal crossing**

crucial mathematical input: [Sabbah,Schnell] to appear

$$\mathfrak{q}_{\mathrm{min}} \sim \exp\left(-\alpha \, \triangle\right)$$

$$\alpha \geq \frac{1}{\sqrt{D-2}}\Big|_{D=4}$$

as required by Emergent String Conjecture

[Lee,Lerche,TW'19]
[Etheredge, Heidenreich,Kaya,Qiu,Rudelius '22]
[Agmon,Bedroya,Kang,Vafa'22]
[Monnee,TW,Wiesner'25]

N=2 SUGRA vs Quantum Gravity

Which necessary extra conditions imposed by Quantum Gravity?

i) **Completeness and existence of particle towers**

Dirac quantisation $\Longrightarrow$ integral Hodge structure $H_{\mathbb{C},t} = H_{\mathbb{Z},t} \otimes \mathbb{C}$

ii) Well defined **duality frames at infinity** **[Kaufmann,TW to appear]** **[Monnee,TW,Wiesner'25]**

$\Longrightarrow$ Simple normal crossing condition

N=2 SUGRA vs Quantum Gravity

[Kaufmann,TW to appear]

Which necessary extra conditions imposed by Quantum Gravity?

Swampland Principle 1. *The vector multiplet moduli space of a 4d $\mathcal{N}=2$ supergravity theory consistent with quantum gravity must admit an integral variation of polarised Hodge structure.*

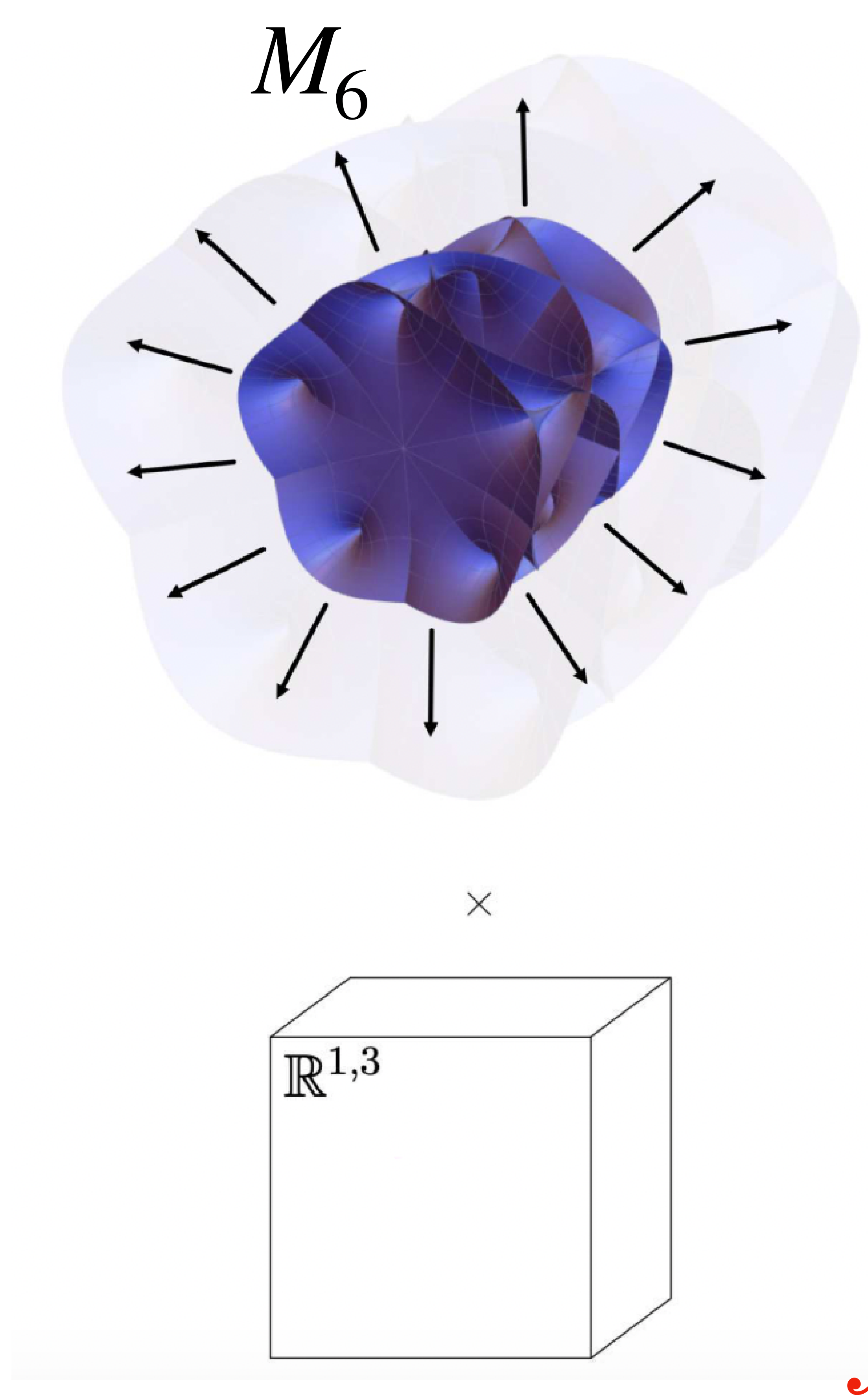
Swampland Principle 2. *The vector multiplet moduli space M_n of a 4d $\mathcal{N}=2$ supergravity coupled to n vector multiplets consistent with quantum gravity can be compactified to a variety $\overline{M}_n$ such that $\overline{M}_n \setminus M_n$ has only simple normal crossing singularities.*

related, but not identical: [Cecotti 2020]

Part II:

Specialization to CY 3-folds (complex structure moduli)

4d N=2 moduli spaces



String theory on $\mathbb{R}^{1,3} \times M_6$

M_6 : Calabi-Yau 3-fold

$h^{1,1}(M_6)$ Kähler moduli

$h^{1,2}(M_6)$ complex structure moduli

$\longleftrightarrow$

massless scalar fields

φ^i in effective action on $\mathbb{R}^{1,3}$

$$\mathscr{M}_{M_6} = \mathscr{M}_{\text{Kahler}} \times \mathscr{M}_{\text{compl.str.}} \longleftrightarrow \mathscr{M} = \mathscr{M}_V \times \mathscr{M}_H$$

of 4d N=2 SUGRA

Type IIB string theory on CY 3-fold X_3

$$S_{4\mathrm{d}} = \int_{\mathbb{R}^{1,3}} \left(\frac{M_{\mathrm{Pl}}^2}{2} \star R - 2g_{i\bar{j}} \mathrm{d}t^i \wedge \star \mathrm{d}\bar{t}^{\bar{j}} + \frac{1}{2} \mathrm{Im}\mathcal{N}_{IJ}(t,\bar{t})\mathcal{F}^I \wedge \star \mathcal{F}^J + \frac{1}{2} \mathrm{Re}\mathcal{N}_{IJ}(t,\bar{t})\mathcal{F}^I \wedge \mathcal{F}^J \right)$$

$I : 0, \quad 1, \ldots, n$

$$t^i = \frac{Z^i}{Z^0}$$

t^i : complex scalar fields (vector multiplet moduli)

graviphoton

U(1) gauge fields A^i

$$t^i := \frac{\int_{\gamma^i} \Omega}{\int_{\gamma^0} \Omega} =: \frac{\log z^i}{2\pi\sqrt{-1}} \qquad A^I = \int_{\gamma^I} C_4 \qquad i = 1, \ldots, n, \qquad n = h^{2,1}(X_3)$$

$$\Pi = \begin{pmatrix} Z^I \\ F_I \end{pmatrix} \qquad Z^I = \int_{\gamma^I} \Omega \qquad F_J = \int_{\gamma_J} \Omega \qquad \gamma^I \cap \gamma_J = \delta^I_J, \qquad \gamma^I, \gamma_J \in H^3(X_3, \mathbb{Z})$$

Complex Structure Limits

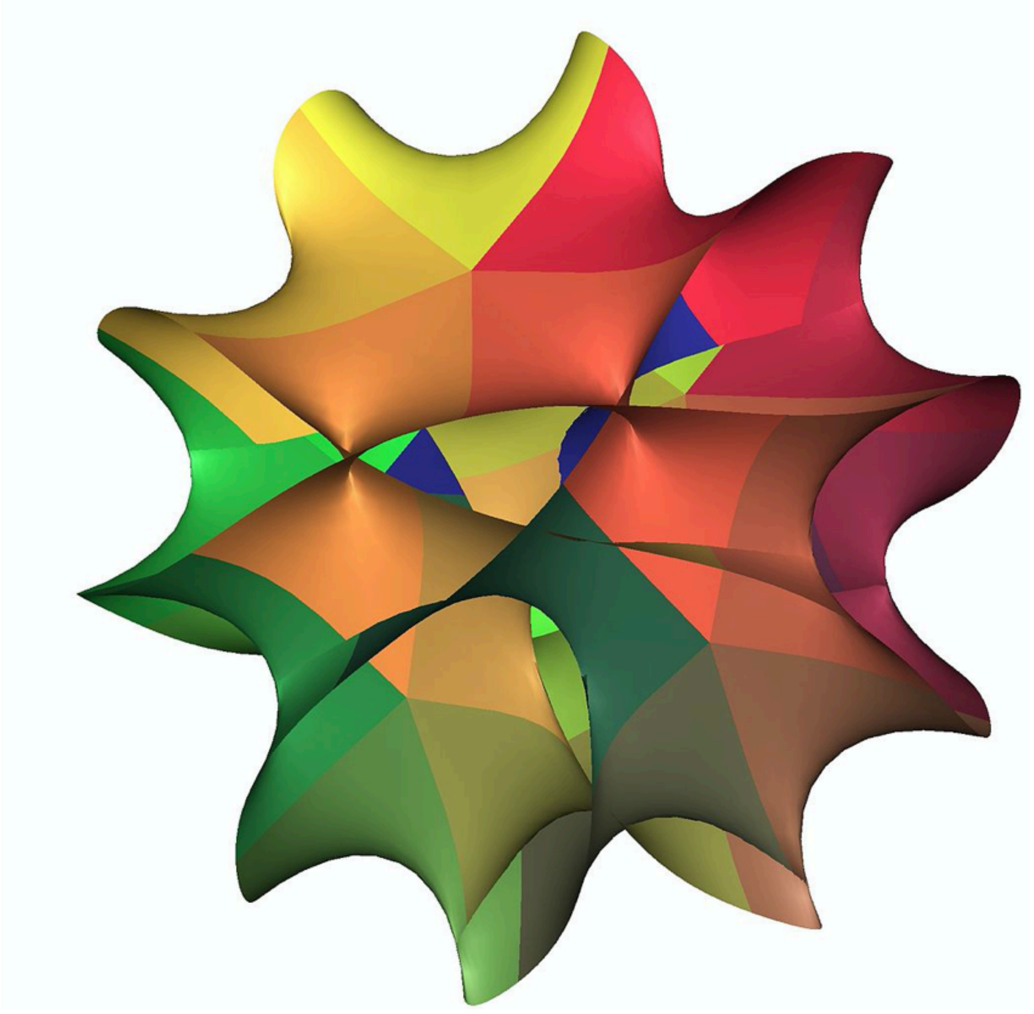
Key question of this talk:

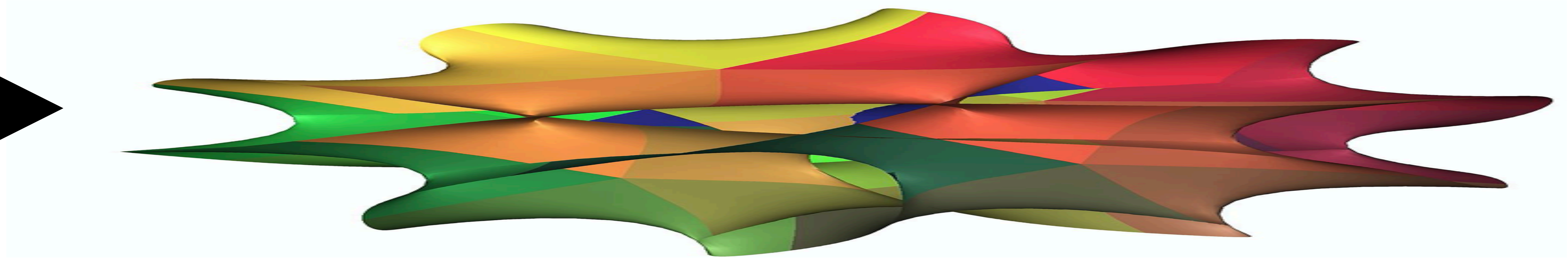
Understand **infinite distance limit** in **complex structure moduli**

at fixed Kähler moduli in **4d N=2 setting**:

$\Longrightarrow$ highly anisotropic space with **large and small 3-cycles**

$$t^i := \frac{\int_{\gamma_i}\Omega}{\int_{\gamma_0}\Omega} =: \frac{\log z^j}{2\pi i}$$





General pattern

- Type IIB theory contains D3-branes: 3+1 dimensional objects
- D3-brane on 3-cycle $\Gamma_3 \subset M_6 \implies$ particle of mass $m \sim \mathrm{vol}(\Gamma)$
- As some 3-cycles become small, states become massless.

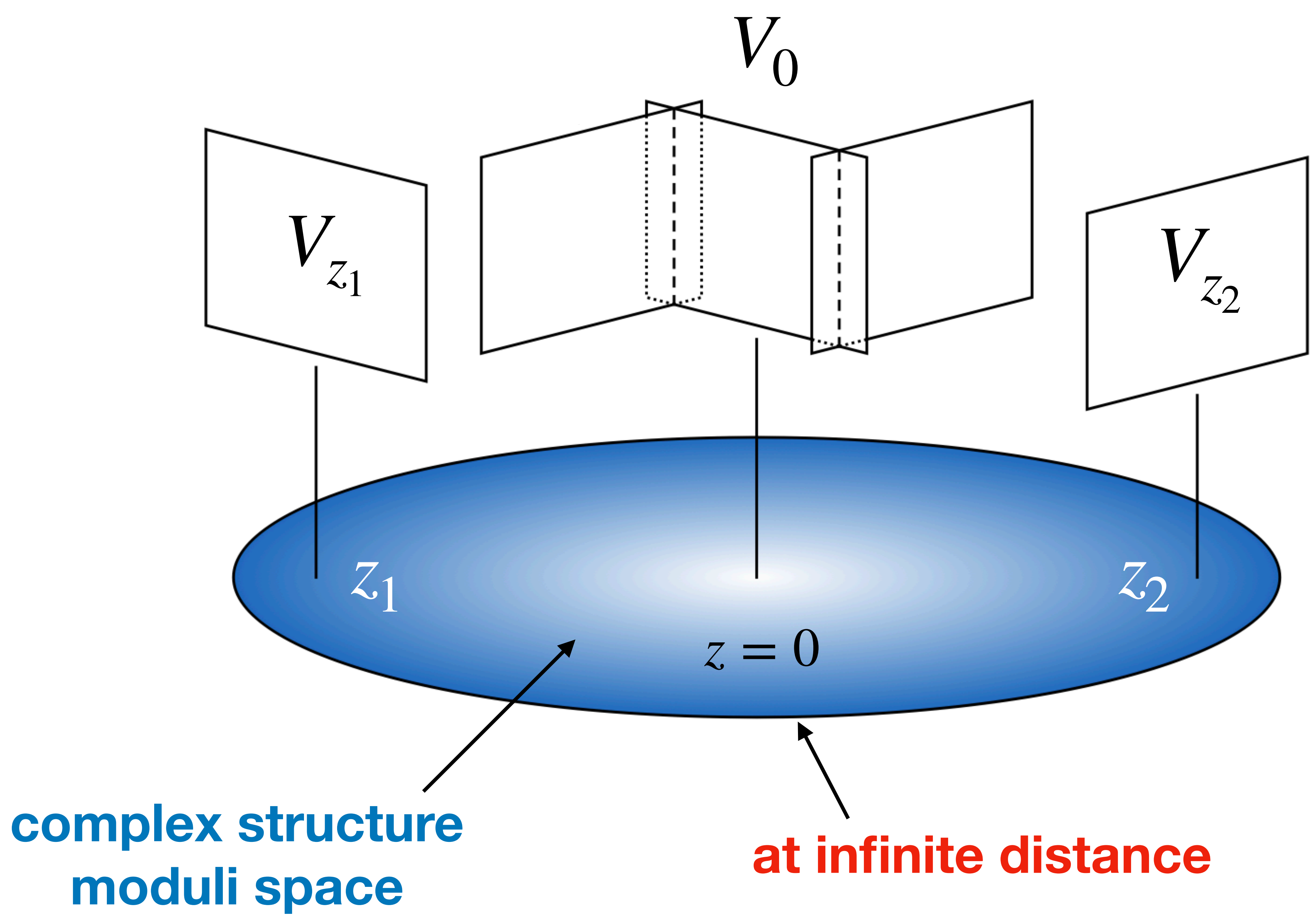
Find:

CY geometry ensures agreement with predictions of

Swampland Distance Conjecture and Emergent String Conjecture

Semi-stable degenerations

CY n-fold $\xrightarrow[\text{degeneration}]{\text{semi-stable}}$ **normally crossing components** [Mumford 1972]



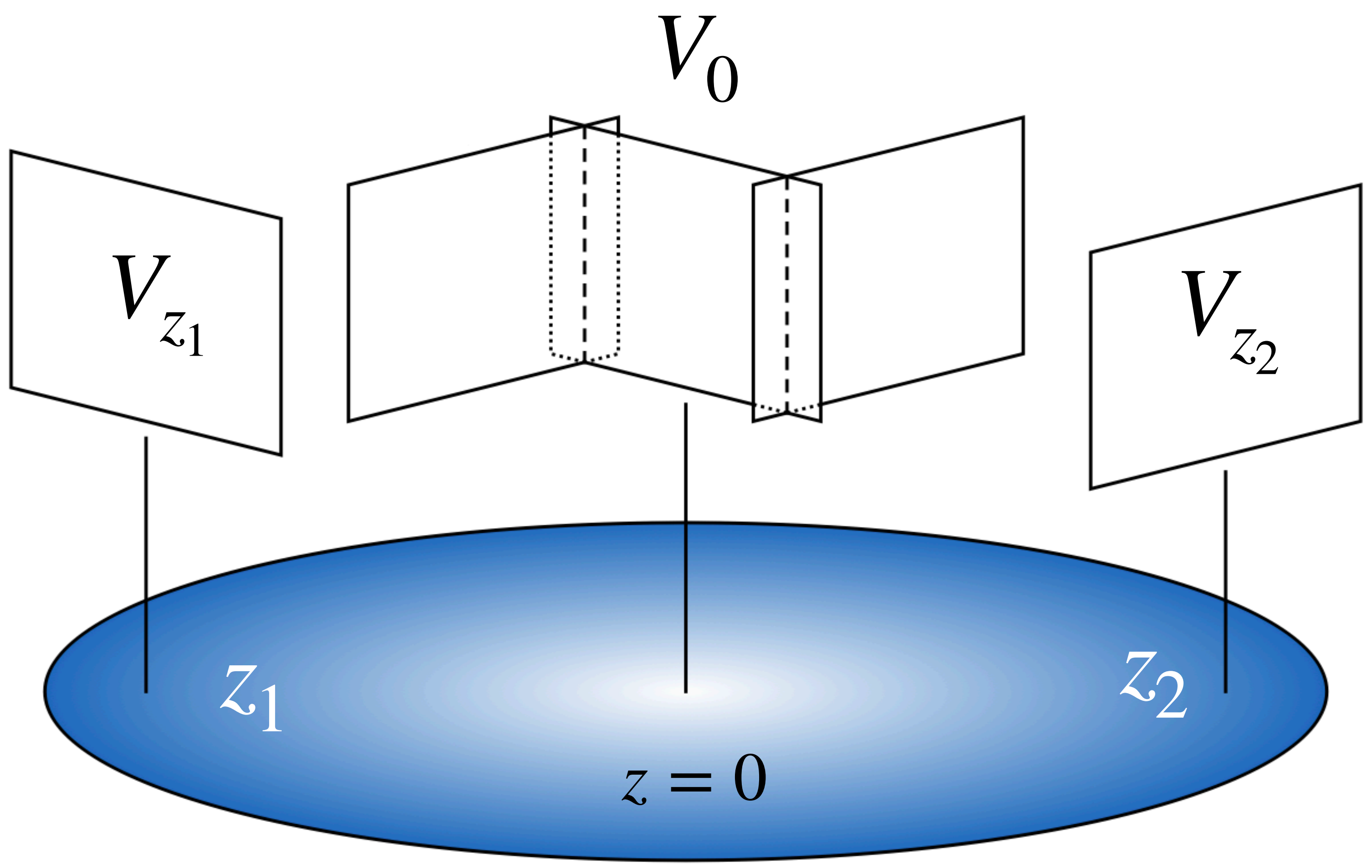
Infinite distance limit

$\Longleftrightarrow$

multiple component degeneration

Semi-stable degenerations

CY3 $\xrightarrow[\text{degeneration}]{\text{semi-stable}}$ **normally crossing components** [Mumford 1972]



$$V \to V_0 = V_1 \cup V_2 \cup \ldots \cup V_n$$

Example:

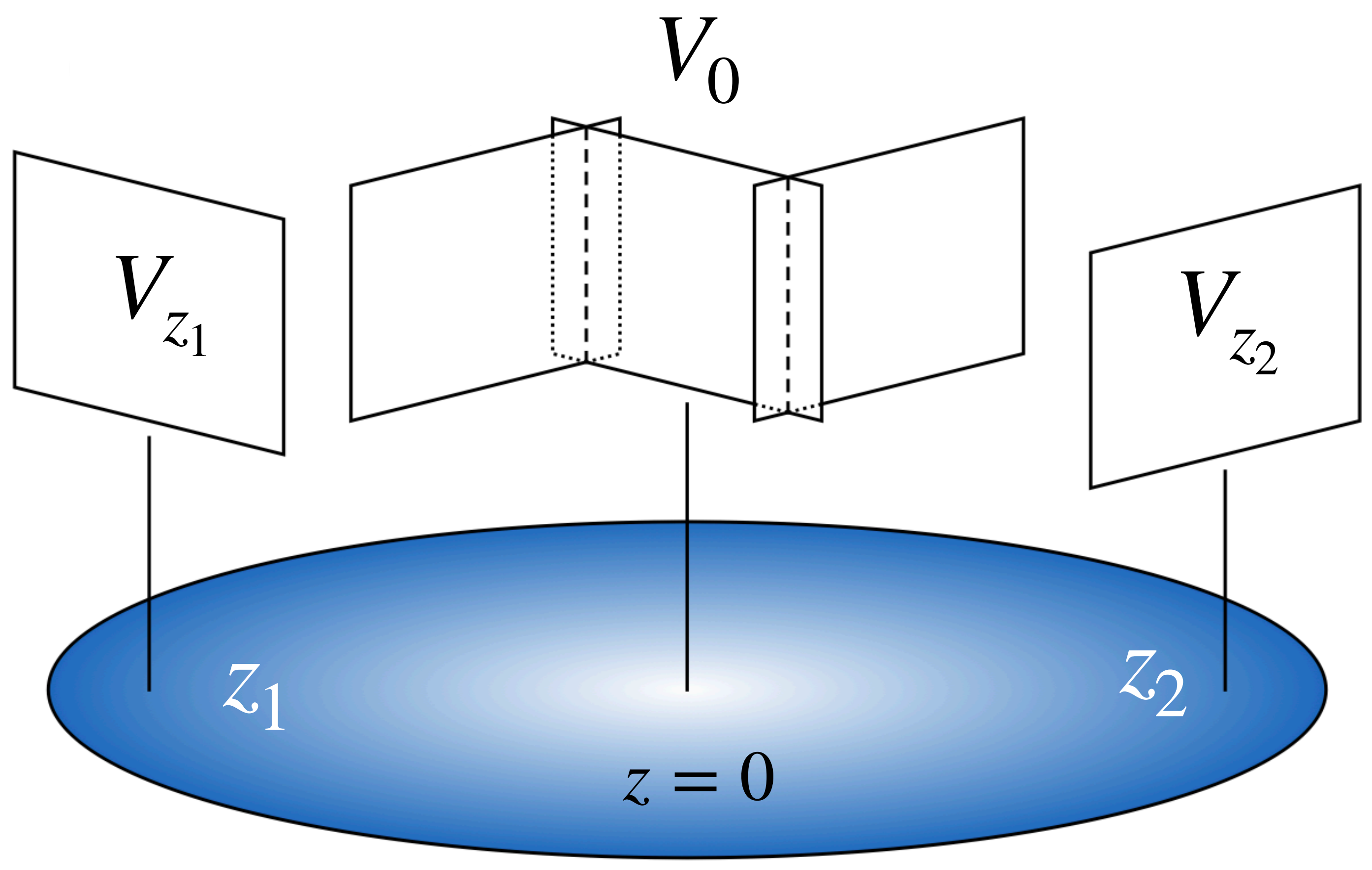
$$P = x_1^{12} + x_2^{12} + x_3^6 + x_4^6 + x_5^2 - u_1^{-1} x_1 x_2 x_3 x_4 x_5 - u_2^{-1} x_1^6 x_2^6 = 0 \quad \xRightarrow{u_2 \to 0} \quad V_0 = V_1 \cup V_2$$

(mod G_{GP})

$$V_1 = \{x_1^6 = 0\}\,, \quad V_2 = \{x_2^6 = 0\}$$

The II, III, IV of infinite distance

CY3 $\xrightarrow[\text{degeneration}]{\text{semi-stable}}$ **normally crossing components**



[Mumford 1972]

$$V_{i_1} \cap V_{i_2} \cap \ldots \cap V_{i_n} = V_{i_1 i_2 \ldots i_n}$$

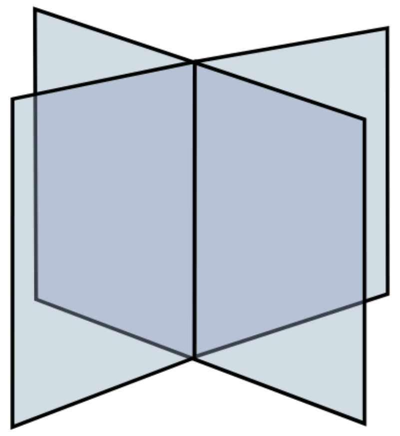
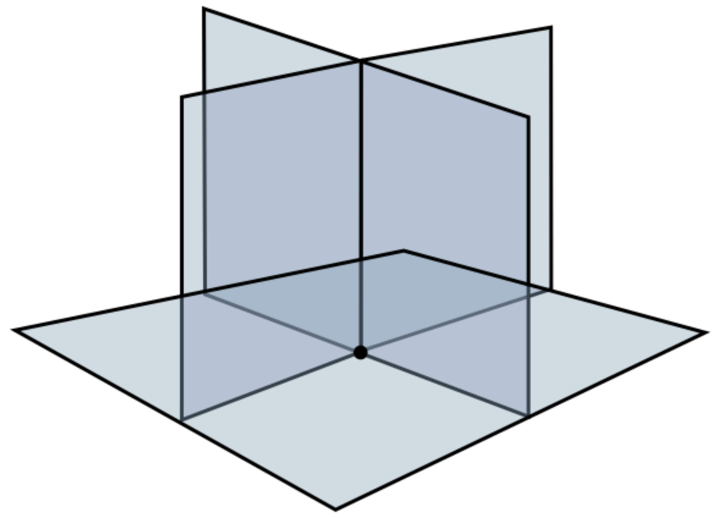
Primary type of degeneration:

Highest complex codimension $d = 1,2,3$

in which components intersect

The II, III, IV of infinite distance

In Hodge theory notation of
[Grimm,Palti,Valenzuela'18]
[Grimm,Li,Palti'18]

Primary type of degeneration:	**Highest complex codimension d of intersection loci:**	[Mumford 1972] … for IIB CY 3-folds: [Hassfeld,Monnee,Wiesner,TW'25] [Monnee,Wiesner,TW'25]	
type II:	$V_{ij} \neq 0$: **double surfaces**, but $V_{ijk} = 0$		$d = 1$
type III:	$V_{ij} \neq 0$: double surfaces, $V_{ijk} \neq 0$: **triple curves**, but $V_{ijkl} = 0$		$d = 2$
type IV:	$V_{ij} \neq 0\,,\quad V_{ijk} \neq 0\,,\quad V_{ijkl} \neq 0$: **quadruple point**		$d = 3$

The II, III, IV of infinite distance

Primary type of degeneration:	**Highest complex codimension d** of **intersection loci:**	**Fastest vanishing 3-cycles Γ_3:**
type II: $d = 1$	$V_{ij} \neq 0$: **double surfaces**, but $V_{ijk} = 0$	fibration
type III: $d = 2$	$V_{ij} \neq 0$: double surfaces, $V_{ijk} \neq 0$: **triple curves**, but $V_{ijkl} = 0$	$T^d \hookrightarrow \Gamma_3 \rightarrow B_{3-d}$
type IV: $d = 3$	$V_{ij} \neq 0\,,\quad V_{ijk} \neq 0\,,\quad V_{ijkl} \neq 0$: **quadruple point**	**real** (3-d) cycle on (3-d) complex dim. intersection locus

[Mumford 1972] ...
for IIB CY 3-folds:
[Hassfeld,Monnee,Wiesner,TW'25]
[Monnee,Wiesner,TW'25]

Vanishing 3-cycles

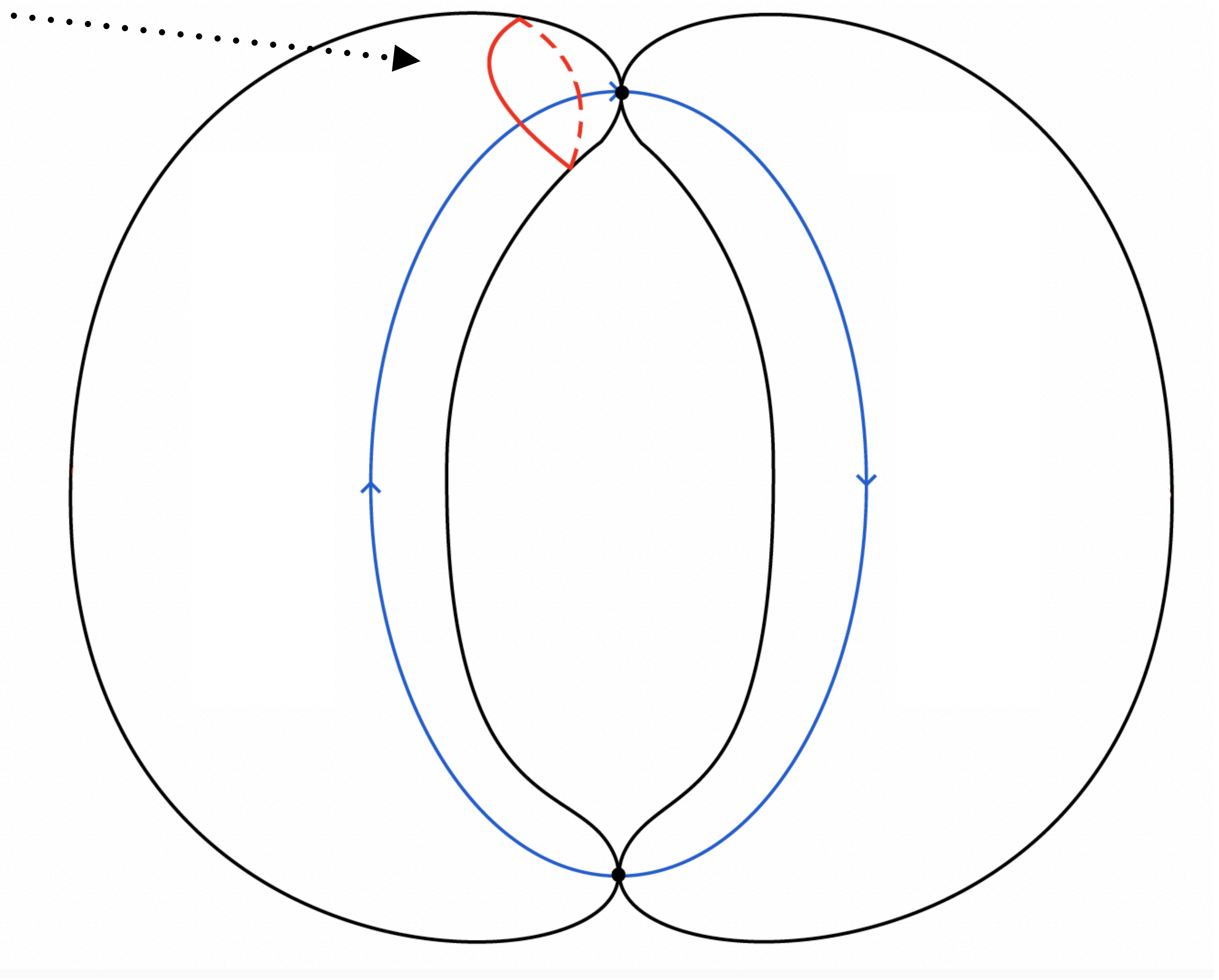
Fastest vanishing 3-cycles:

fibration $T^d \hookrightarrow \Gamma \to B_{3-d}$

real (3-d) cycle on (3-d) complex dim. intersection locus

$\Gamma =$ *vanishing* S^1

Toy example:
Degeneration of T^2



For elliptic K3:
[Lee,Lerche,TW'21]

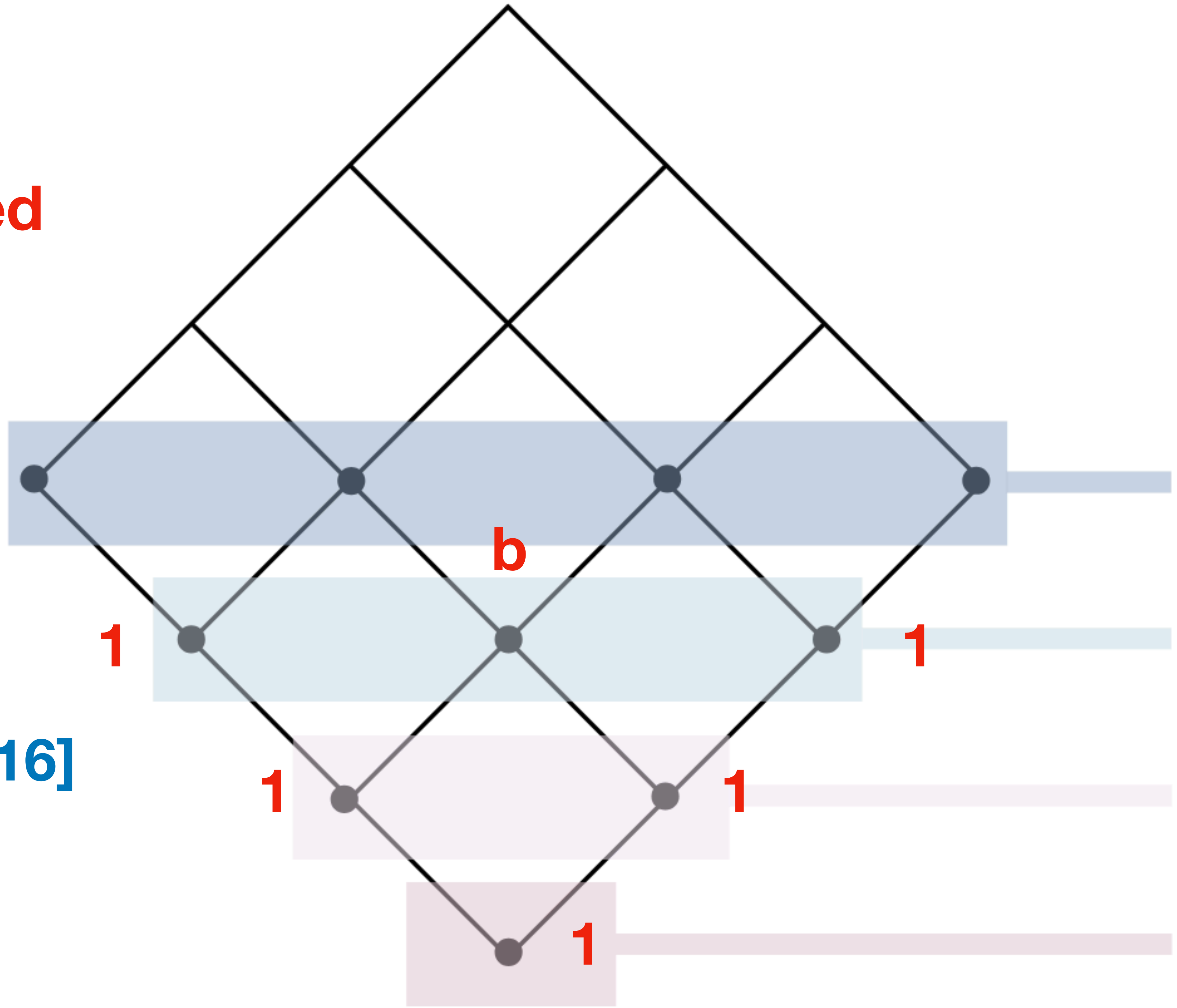
For elliptic CY 3-folds
[Alvarez-Garcia,Lee,TW'23]

Vanishing 3-cycles in type II, III, IV

Fastest vanishing 3-cycles: fibration $T^d \hookrightarrow \Gamma \to B_{3-d}$ ← real (3-d) cycle on (3-d) intersection locus

<u>Geometric</u> limiting mixed Hodge structure

[Deligne '71/'74]
[Morrison'84]
Doran,Thompson,Harder'16]



[Hassfeld,Monnee,TW,Wiesner'25]
[Monnee,TW,Wiesner'25]

type I (finite distance)

type II $\quad S^1 \hookrightarrow \Gamma^a \to C_2^a$

type III $\quad T^2 \hookrightarrow \Gamma^{(A,B)} \to S_1^{(A,B)}$

type IV $\quad \Gamma = T^3$

Vanishing 3-cycles in type II, III, IV

Infinite distance limit in complex structure moduli space of **Type IIB on CY 3-fold V**

$$\frac{\log u^j}{2\pi \mathrm{i}} = a^j + \mathrm{i}s^j \to \mathrm{i}\infty$$

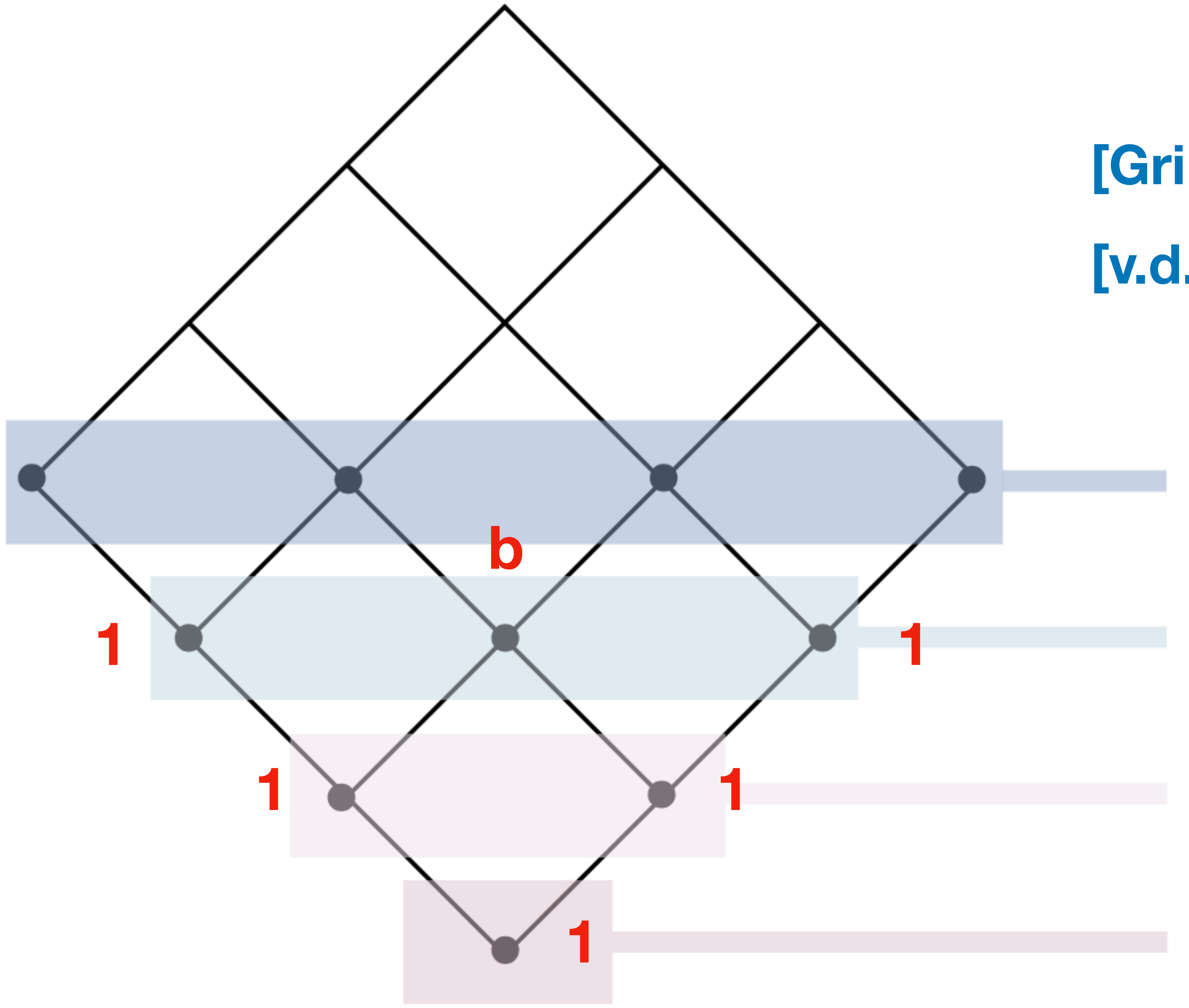
axions saxions

[Grimm,Palti,Valenzuela'18] [Grimm,Li,Palti'18] …

[v.d. Heisteeg'22] [Monnee'24]

Algebraic limiting mixed Hodge structure

$$H^3(V,\mathbb{C}) = \bigoplus_{\ell=0}^{6} \mathrm{Gr}_\ell$$



type I (finite distance)

$\mathrm{vol}(\Gamma) \sim s^{-1}$ type II $S^1 \hookrightarrow \Gamma^a \to C_2^a$

$\mathrm{vol}(\Gamma) \sim s^{-2}$ type III $T^2 \hookrightarrow \Gamma^{(A,B)} \to S_1^{(A,B)}$

$\mathrm{vol}(\Gamma) \sim s^{-3}$ type IV $\Gamma = T^3$

[Cattani,Kaplan,Schmid'86]

Physics interpretation

Infinite distance limit in complex structure moduli space of **Type IIB on CY 3-fold** V

$$\frac{\log z^j}{2\pi i} = a^j + i s^j \to i\infty$$

axions saxions

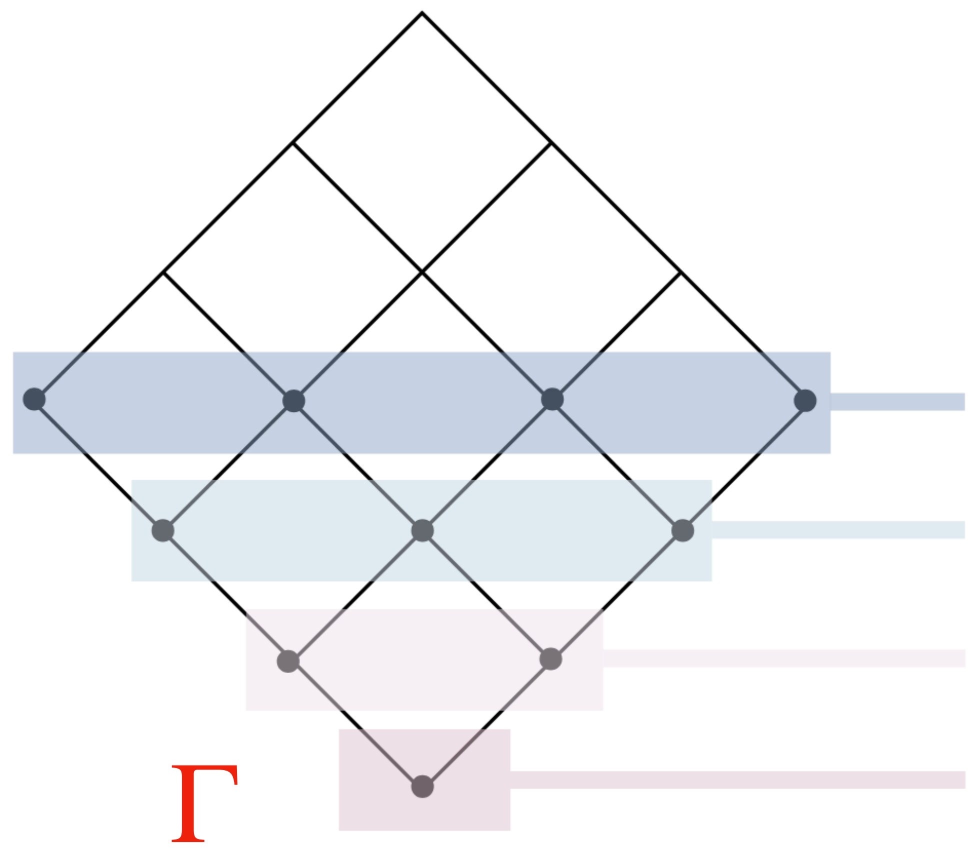
$\mathrm{vol}(\Gamma) \sim s^{-1}$	type II	$S^1 \hookrightarrow \Gamma^a \to C_2^a$	**Emergent string limits**
$\mathrm{vol}(\Gamma) \sim s^{-2}$	type III	$T^2 \hookrightarrow \Gamma^{(A,B)} \to S_1^{(A,B)}$	**Decompactification 4d $\to$ 6d**
$\mathrm{vol}(\Gamma) \sim s^{-3}$	type IV	$\Gamma = T^3$	**Decompactification 4d $\to$ 5d**

Algebraic mixed Hodge structure + **Geometric mixed Hodge structure** $\Longrightarrow$ **Physics**

Type IV as decompactification $4d \to 5d$

1 leading vanishing 3-cycle Γ of topology T^3 $\Longrightarrow$ 1 tower of BPS states from wrapped D3-branes

behaving like 1 KK tower [Monnee,TW,Wiesner'25]



Reason:

$$\boxed{\text{BPS index: } \Omega(n\Gamma) = \Omega(\Gamma)\,, \quad n > 0}$$

From this we can in fact show: $\Omega(\Gamma) = -\chi(V)$

$$\Omega^{(0)}_{\text{BPS,5d}} = -\sum_{j_R} (-1)^{2j_R}(2j_R+1)n^0_{(j_L,j_R)} = -2n^{(0)}_H + 2n^{(0)}_V + 4 \qquad n^{(0)}_H = h^{1,1}(V)+1\,, \quad n^{(0)}_V = h^{2,1}(V)-1$$

Known by mirror symmetry for large complex structure points,

but **true more generally for all type IV degenerations and without using mirror symmetry**!

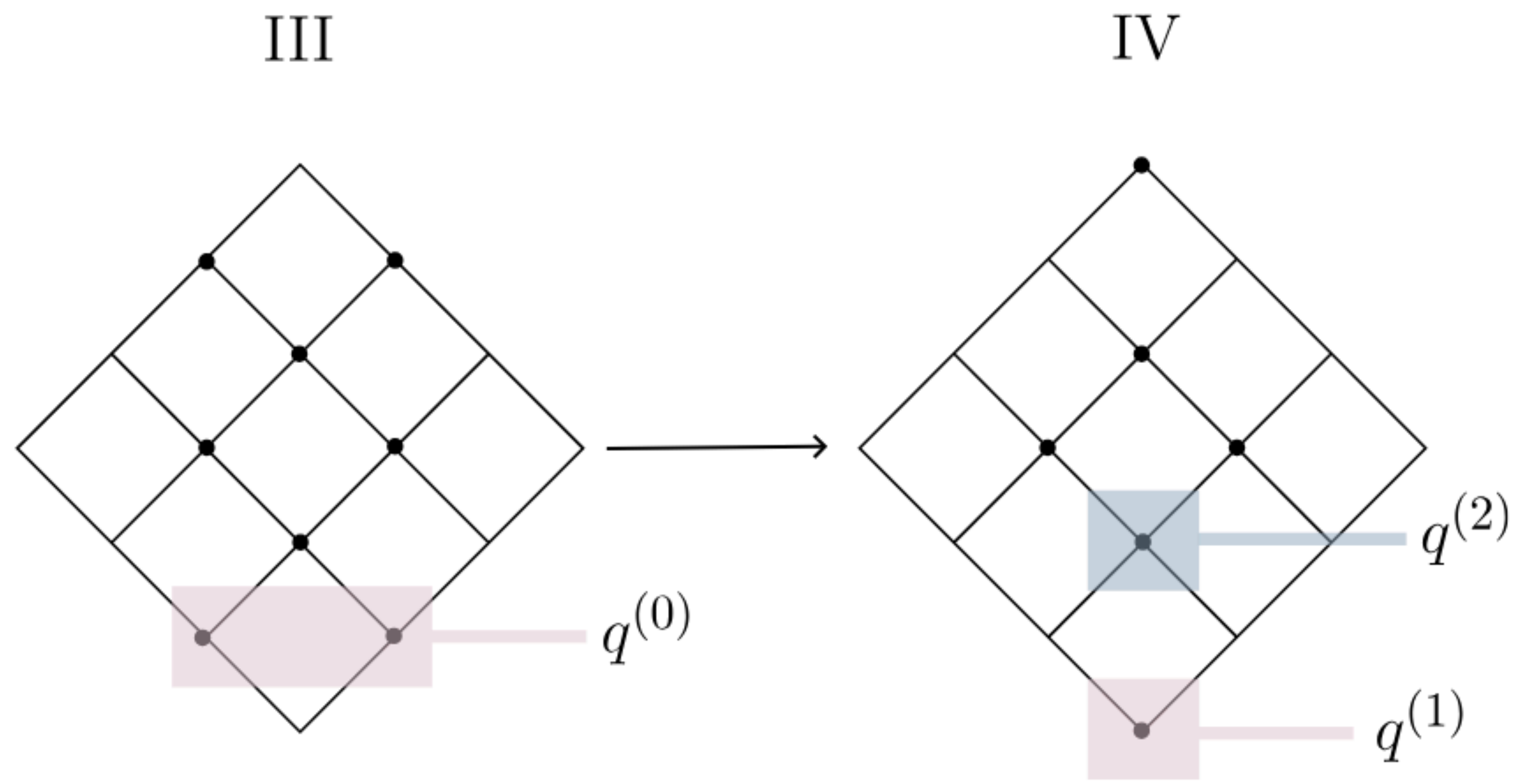
Type III as decompactification $4d \to 6d$

2 fastest vanishing 3-cycles $\Gamma^{A,B}$ of topology T^3 $\Longrightarrow$ 2 towers of BPS states from wrapped D3-branes **behaving as 2 KK towers**

In fact: $\boxed{\text{BPS index: } \Omega(n\Gamma^{(i)}) = \Omega(\Gamma^{(i)}) = -\chi(V)\,, \quad n > 0 \quad i = A, B}$

[Monnee,TW,Wiesner'25]

By noting that
enhancement
type III to IV
is always possible



Type II as emergent string limit - Upshot

1) Identify tensionless SUGRA BPS string:

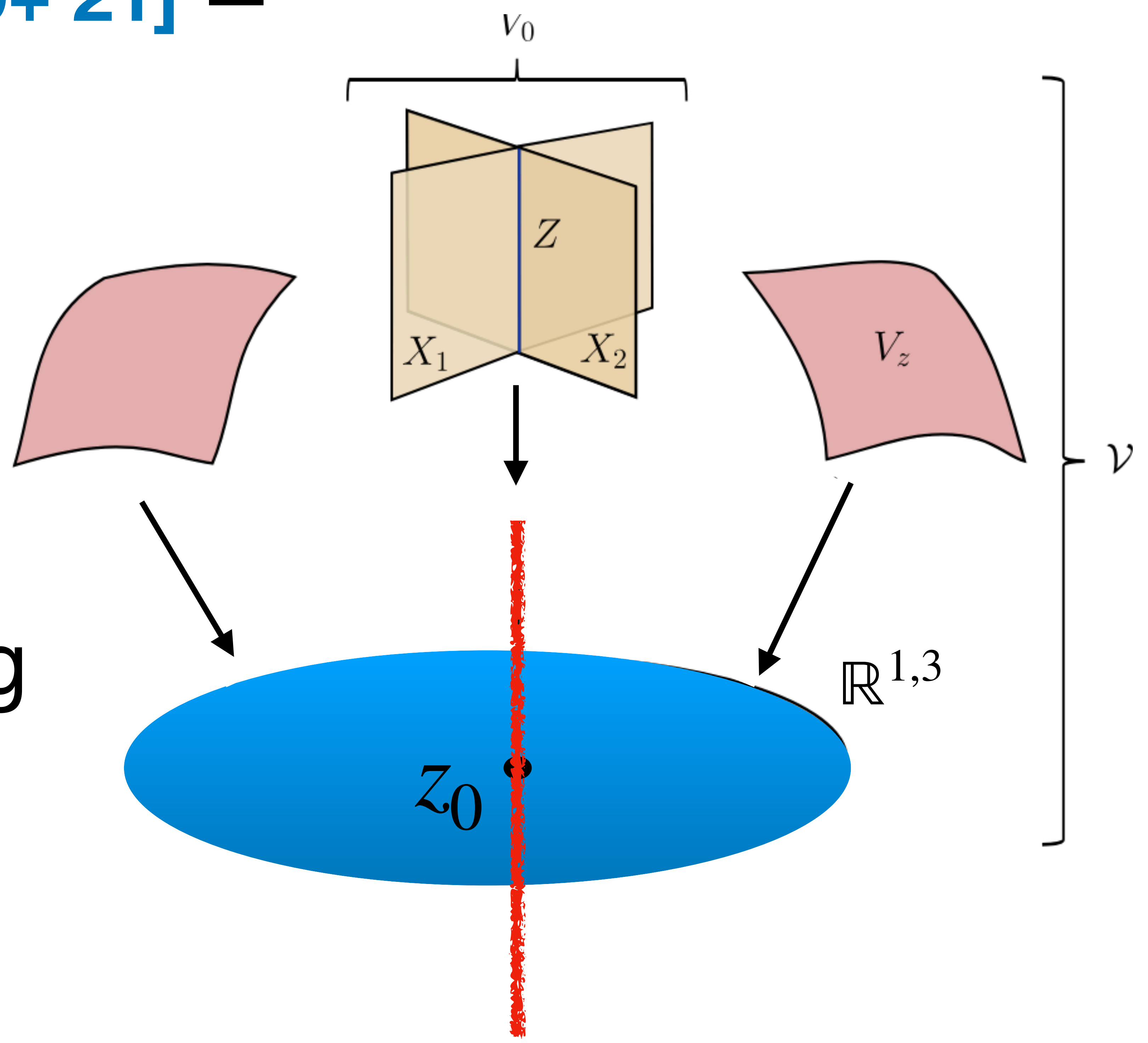
— EFT string of [Lanza,Marchesano,Martucci,Valenzuela'20+'21] —

as a **critical heterotic or Type II string**

Key: V_{12} is K3 or T^4

2) D3 branes on $\Gamma^{(a)}$: Dual winding states of that string

Details: [Hassfeld,Monnee,Wiesner,TW'25]



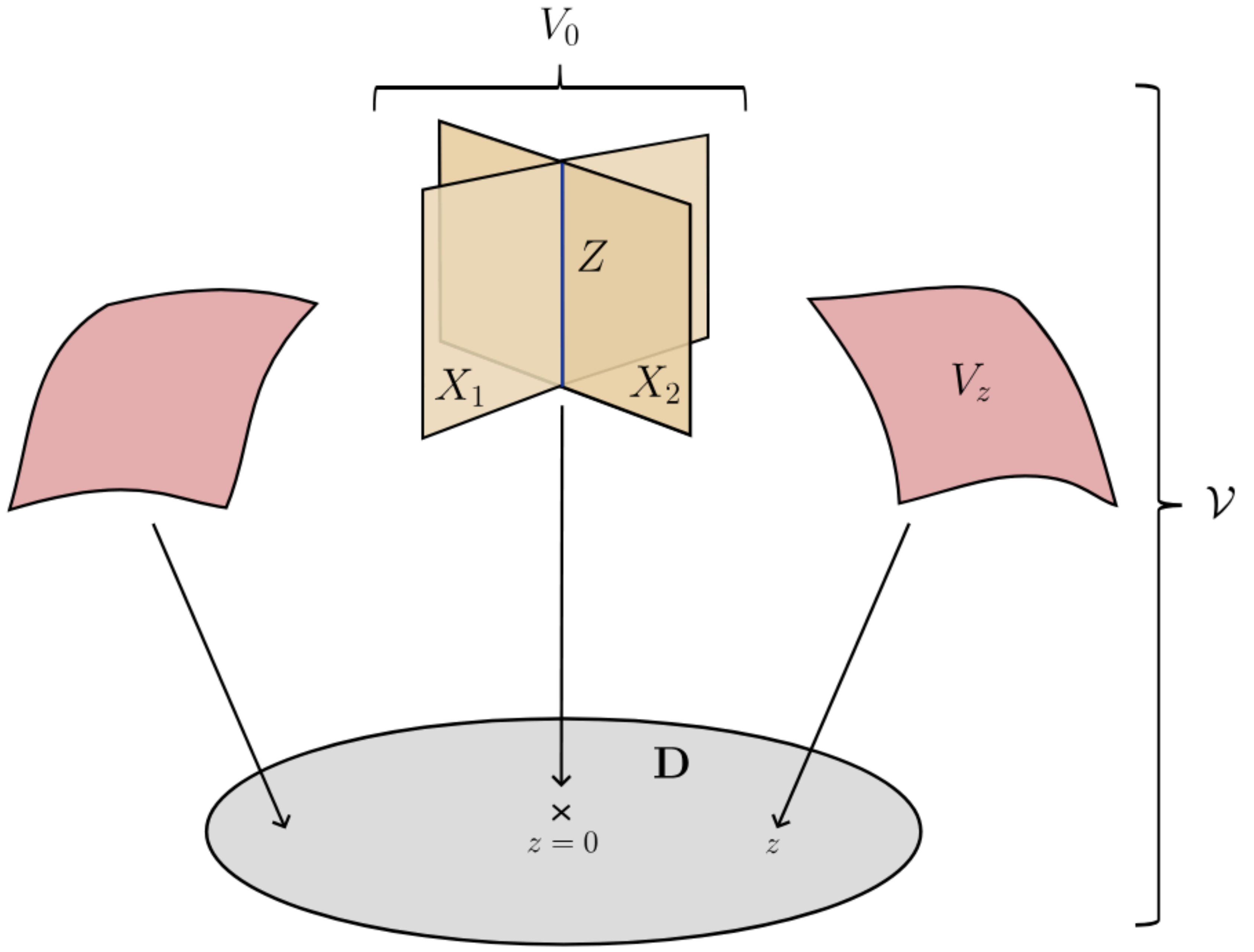
Type II: Tyurin degenerations

Special class of Type II degenerations:

Tyurin degenerations

$V_0 = X_1 \cup_Z X_2$

Z : **K3 surface**



$$H^2(Z) = \Lambda_{\text{pol}} \oplus \Lambda_{\text{trans}}$$

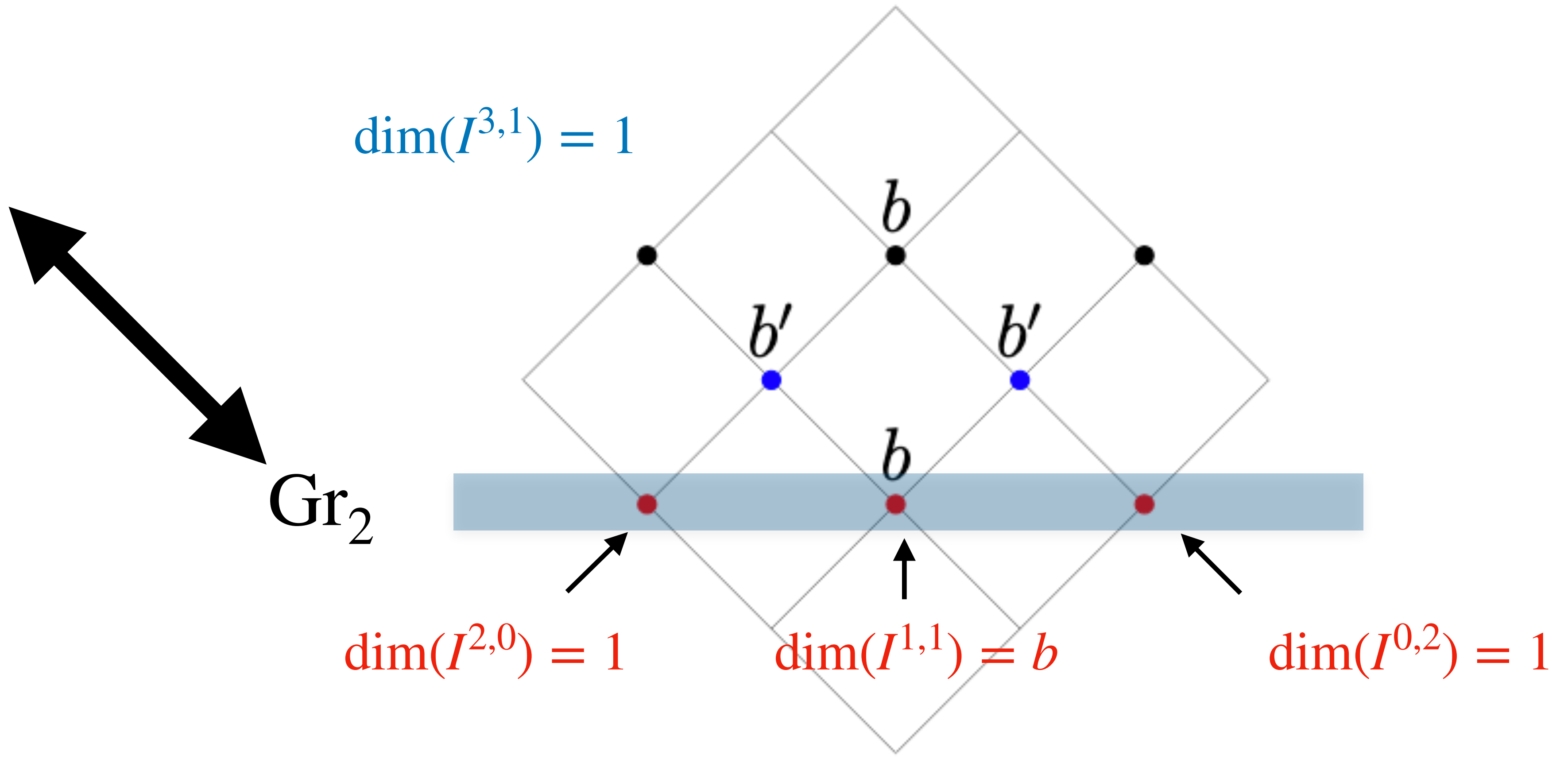
[Deligne '71/'74]
[Morrison'84]
Doran,Thompson,Harder'16]

$i^*(H^2(X_1)) \oplus j^*(H^2(X_2))$

signature $(1,19-b)$

$H^2(Z)/(\text{im}(i^*) + \text{im}(j^*)) \;\; \cong \text{Gr}_2$

signature $(2,b)$ **for** II_b



$i : Z \hookrightarrow X_1\,, \qquad j : Z \hookrightarrow X_2$

BPS towers from D3-branes

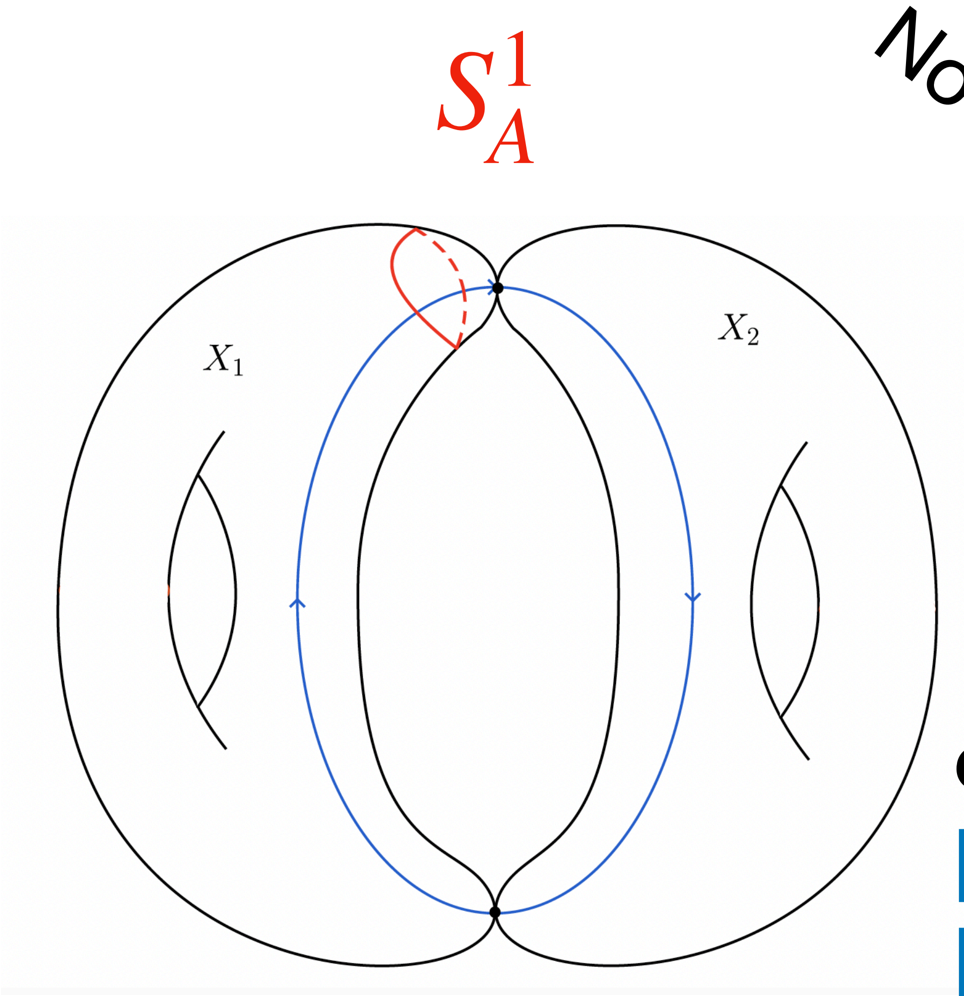
Candidates for BPS towers from wrapped D3-branes:

3-cycles Γ_a dual to $q \in H^3(V, \mathbb{Z}) \cap \mathrm{Gr}_2$

$$\frac{M^2}{M_{\mathrm{Pl}}^2} = \|q\|^2 \sim \frac{1}{s} \to 0$$

$\Updownarrow$

3-forms completed from 2-forms in Λ_{trans}(Z)
by leg in normal direction



Note: S_A^1 monodromy invariant!

$\Longrightarrow$ Dual 3-cycle Γ^a **locally fibration**: $\quad S_A^1 \hookrightarrow \Gamma_a \to C_a$

cf.
[Lee,Lerche,TW'21]
[Alvarez-Garcia,Lee,TW'23]

- Wrapped D3 brane BPS if Γ_a special Lagrangian: $\quad J|_{C_a} = 0$

  $\Longleftrightarrow$ C^a transcendental on Z ✔

- Multi-wrapping possible for $b_1(\Gamma_a) \geq 3$ i.e. $g(C_a) \geq 1$ $\quad \Longleftrightarrow \quad C_a \cdot_Z C_a \geq 0$

BPS towers from D3-branes

Tower from D3-branes: 3-cycles Γ_a : $S^1_A \hookrightarrow \Gamma_a \to C_a$ dual to $q \in H^3(V, \mathbb{Z}) \cap \mathrm{Gr}_2$ $\qquad \frac{M^2}{M^2_{\mathrm{Pl}}} = \|q\|^2 \sim \frac{1}{s} \to 0$

with $C_a \cdot_Z C_a \geq 0$ transcendental 2-cycle on Z

Interpretation of tower: KK and winding states of dual (emergent) heterotic string along T^2_{het}:

$$C_a \in \Lambda_{\mathrm{trans}}(Z) = U_{1,1} \oplus U_{1,1} \oplus \Gamma_{0,b-2}$$

Conjecture: [Hassfeld,Monnee,TW,Wiesner'25]

BPS indices for tower counted by coefficients of an - in general mock - modular form:

$$\theta(q) = \sum_{n \in \mathcal{I}} c(n) q^n : \qquad \Omega_{\mathrm{BPS}}(\Gamma_a) = c\left(\frac{1}{2} C_a \cdot_Z C_a\right)$$

cf. [Banerjee,DiPietro,Longhi'22]

motivated by known counting in IIA-heterotic duality of Harvey-Moore states

Conclusions

Calabi-Yau geometry knows about key properties that are believed to be deeper, general features of **Quantum Gravity.**

Must disentangle:
Consequences of supergravity alone vs. actual properties of QG - as done in N=2 here.

Next steps: Reduce supersymmetry

$\mathcal{N} = 1$: **Important modifications (quantum obstructions)** **[Kaufmann,Monnee,TW,Wiesner'26]** **[Kaufmann,TW,Wiesner'26]**

Again:
Special holonomy geometry (partially) guides the way - e.g. elliptic Calabi-Yau 4-folds!

Happy 50 th anniversary,

dear Yau's theorem,

and

Congratulations, Professor Yau!

Appendix

A new conjecture for type II limits

All type II limits should be emergent string limits:

$\Lambda_s \sim \sqrt{T_{\text{EFT}}} \sim M_{\text{tower}}$ as computed algebraically

[Dvali'07] [Dvali,Lüst'11]
[Lanza,Marchesano,Martucci,Valenzuela'20+'21]
[Heisteeg,Vafa,Wiesner,Wu'22] [Cribiori,Lüst,Staudt'22]
[Martucci,Risso,Valenti,Vecchi'24]

Conjecture: Generalised Tyurin degenerations [Hassfeld,Monnee,TW,Wiesner'25]

i) *For every type II_b limit **of a CY 3-fold***

$V_0 = \cup_i X_i$ *w/* X_i *intersecting over* $\left\{ \begin{array}{l} \textit{K3 surface} \Longrightarrow \textit{heterotic} \\ T^4 \textit{ surface} \Longrightarrow \textit{Type II} \end{array} \right\}$ *emergent string*

ii) *In particular: parameter* $b \leq 19$