Exploring the Dictionary between Particle Physics and Calabi–Yau Geometry

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BIMSA Member Seminar

遂古之初，谁传道之？
上下未形，何由考之？

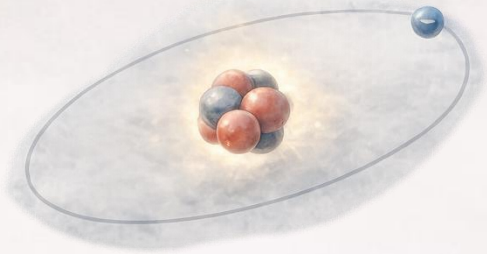
屈原 《天问》

At the beginning of deep antiquity, who passed down the tale?
When heaven and earth had not yet taken form, how could they be examined?

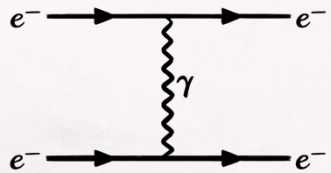
Qu Yuan, Questions to Heaven

What particle physics is looking for?

What are the fundamental building blocks of matter?



How do these building blocks interact?



What is the ultimate structure of nature?

The Standard Model of Particle Physics

FERMIONS

Quarks

1st generation	u	d
2nd generation	c	s
3rd generation	t	b

Leptons

1st generation	e	ν_e
2nd generation	μ	ν_μ
3rd generation	τ	ν_τ

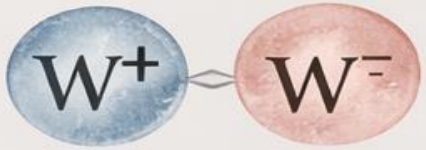
GAUGE BOSONS



g (×8)
Strong Interaction



Photon
Electromagnetic Interaction



W^+ W^-
Charged Weak Interaction



Z^0
Neutral Weak Interaction

SCALAR BOSON



H
Higgs Boson

Theoretical framework of particle physics

Gauge symmetry		$SU(3) \times SU(2) \times U(1)$
Matter fields		fundamental reps of the gauge symmetry group
Gauge interactions		Determined by gauge symmetry
Yukawa coupling		Coupling between matter fields and the Higgs field

Why look beyond the Standard Model?

The Standard Model is extremely successful in describing known particle interactions. However, several fundamental questions remain open:

- It lacks a consistent incorporation of gravity,
- Origin of gauge structure and matter content,
- The hierarchy and naturalness problems,
- Unification of interactions.



N=1 supersymmetry

These questions motivate the search for a more fundamental framework.

String theory, a strong candidate for unification

String theory provides a framework that can consistently unify quantum field theory with gravity.

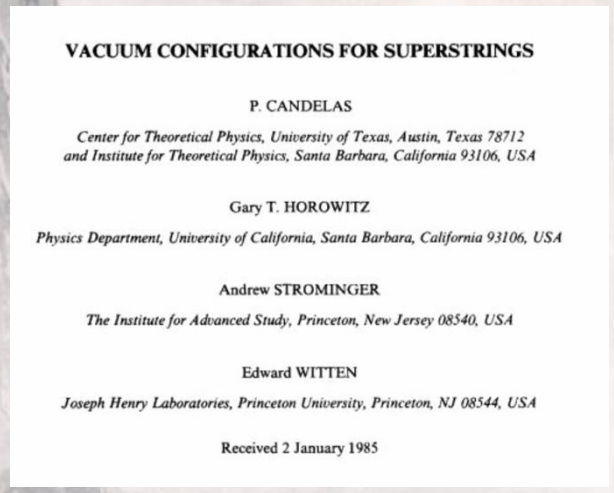
- The fundamental objects are **one-dimensional strings** rather than point particles. Different vibrational modes of a string correspond to **different particles**

- In particular, the spectrum naturally contains a **massless spin-2 particle (the graviton)**, providing a quantum theory of gravity

- There are five consistent ten-dimensional string theories: Type I, Type IIA, Type IIB, heterotic $SO(32)$, and heterotic $E_8 \times E_8$. My work focuses on the **heterotic** $E_8 \times E_8$ theory, since it naturally accommodates realistic gauge groups relevant for particle physics.

A consistent string theory lives in 10-dimensional spacetime. So what is the extra inner space?

Calabi-Yau manifolds



VACUUM CONFIGURATIONS FOR SUPERSTRINGS

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Received 2 January 1985



Proc. Natl. Acad. Sci. USA
Vol. 74, No. 5, pp. 1798–1799, May 1977
Mathematics

Calabi's conjecture and some new results in algebraic geometry

(Kähler manifold/Chern class/Ricci tensor/complex structure)

SHING-TUNG YAU

Mathematics Department, Stanford University, Stanford, California 94305

Communicated by S. S. Chern, January 31, 1977

Preserving $N = 1$ supersymmetry requires

$$M_{10} = M_4 \times X_6$$

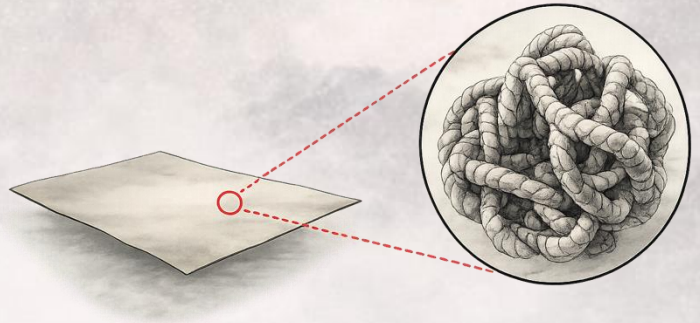
where the internal space X_6 must be a Kähler manifold with a Ricci-flat metric.

The existence of such manifolds was conjectured by Calabi (1957) and proved by Yau (1977)

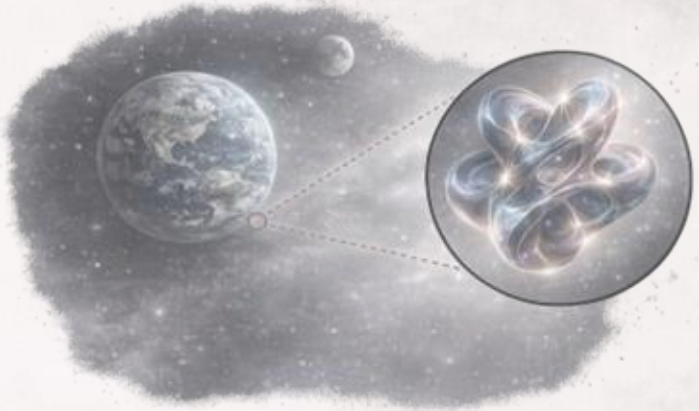
This result plays a central role in connecting string theory with geometry and provides the geometric setting for string compactifications.

Intuitive introduction to compactification

in superstring theory, spacetime is **ten-dimensional**. Why do we observe only four dimensions? A simple analogy is a **sheet of paper**: from far away it appears two-dimensional, but close inspection reveals additional structure.



Similarly in string theory, the extra six dimensions can be compactified on a small internal space. If we have high energy probes, this small compactified space can be detected as well.



The geometry of the compact space strongly influences the resulting four-dimensional theory. Geometry becomes a physical input, not just a mathematical background.

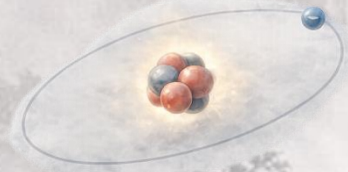
A geometry-physics dictionary

Compactification on a **Calabi–Yau manifold** creates a correspondence between **geometry** and **particle physics**.



Initial data in a heterotic string compactification: a Calabi-Yau manifold X, a stable holomorphic SU(n) vector bundle V satisfy certain constraints

Geometry side		Physics side
E(8)	→ E(6), SO(10), SU(5) →	Standard model gauge group
Bundle cohomology $H^1(X,V)$ …	→	Family number of particles
Triple product	→	Yukawa coupling



Questions in particle physics can often be reformulated as questions in geometry. Conversely, classification problems in geometry can have direct physical interpretation. This interplay is one of the main reasons Calabi-Yau geometry remains central in string phenomenology and mathematical physics.

Ultimate objective of this geometry-physics dictionary

Compactification is **not arbitrary**: different internal geometries lead to different four-dimensional effective theories.

Therefore constructing **Standard Model–like theories** in string theory requires two steps:



Enumerate possible Calabi–Yau threefolds



Construct particle physics models on these manifolds

The number of possible compactification geometries is extremely large. Even within specific classes such as **complete intersection Calabi–Yau manifolds (CICYs)**, the structure is already very rich.

This naturally leads to **large-scale classification and search problems.**

Calabi-Yau manifolds construction in our dictionary

Toric Calabi-Yau manifolds

- Constructed as hypersurfaces (or complete intersections) in toric varieties determined by reflexive polytopes. Total number of them is more than 4.7×10^8 . Batyrev (1993), Batyrev and Borisov (1994), Kreuzer and Skarke (2000),

Complete Intersection Calabi-Yau Manifolds (CICY)

- Realized as complete intersections of homogeneous polynomials in products of projective spaces. Total number of topologically distinct CICY is 7890. Candelas, Dale, Lütken & Schimmrigk (1988), Green, Hübsch and Lütken (1989),

$$X = \left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & q_1^1 & \cdots & q_1^K \\ \mathbb{P}^{n_2} & q_2^1 & \cdots & q_2^K \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & q_m^1 & \cdots & q_m^K \end{array}\right] \qquad \sum_{a=1}^{K} q_r^a = n_r + 1, \qquad r = 1, \ldots, m$$

Generalized CICY

- Generalized CICYs extend the CICY construction by allowing defining sections of line bundles that are not globally generated on the ambient space but exist on intermediate subvarieties. Anderson et al (2016)

Model building in the dictionary

First model constructed by Candelas, Horowitz, Strominger and Witten in 1985, by using the tangent bundle of Calabi-Yau as the vector bundle

Braun, He, Ovrut, Pantev (2005) first got ONE realistic standard model in heterotic string compactification, which can reproduce exact gauge group and matter content and soon in 2006 they got another model without exotic particles.

Bouchard, Donagi (2006) got ONE explicit standard model in heterotic string compactification, with exact gauge group and matter content without exotic particles

Blumenhagen, Moster, Weigand (2006) first did a systematic scan which resulted in 8 explicit constructions.

Anderson, Gray, He, Lukas (2010) did a large-scale algorithmic scans produced 91 consistent heterotic models, but **only one fully explicit MSSM construction** was obtained.

Anderson, Gray, Lukas and Palti (2012) found the first **large, explicitly constructed data sets of heterotic MSSM vacua** on smooth compact Calabi–Yau spaces, marking a clear shift from isolated classic examples to large-scale systematic model building in the heterotic landscape

When we dive deeper into the dictionary

Calabi-Yau manifolds have complex structure moduli. Different choices of these moduli correspond to different **geometric shapes** of the same manifold.

An important fact is that **the cohomology groups that determine the chiral spectrum are moduli dependent**, and their dimensions can jump on special loci in moduli space.

In **Gray and Wang (2019)** we showed that the four–dimensional particle spectrum of heterotic line bundle models on smooth Calabi–Yau threefolds can **abruptly change ("jump")** as one moves in complex–structure moduli space.

Therefore, the **"shape" of the Calabi–Yau manifold plays a crucial role in string model building.**



Complex structure moduli

For consistency, **model building and moduli stabilization must be studied simultaneously.**

When we dive deeper into the dictionary

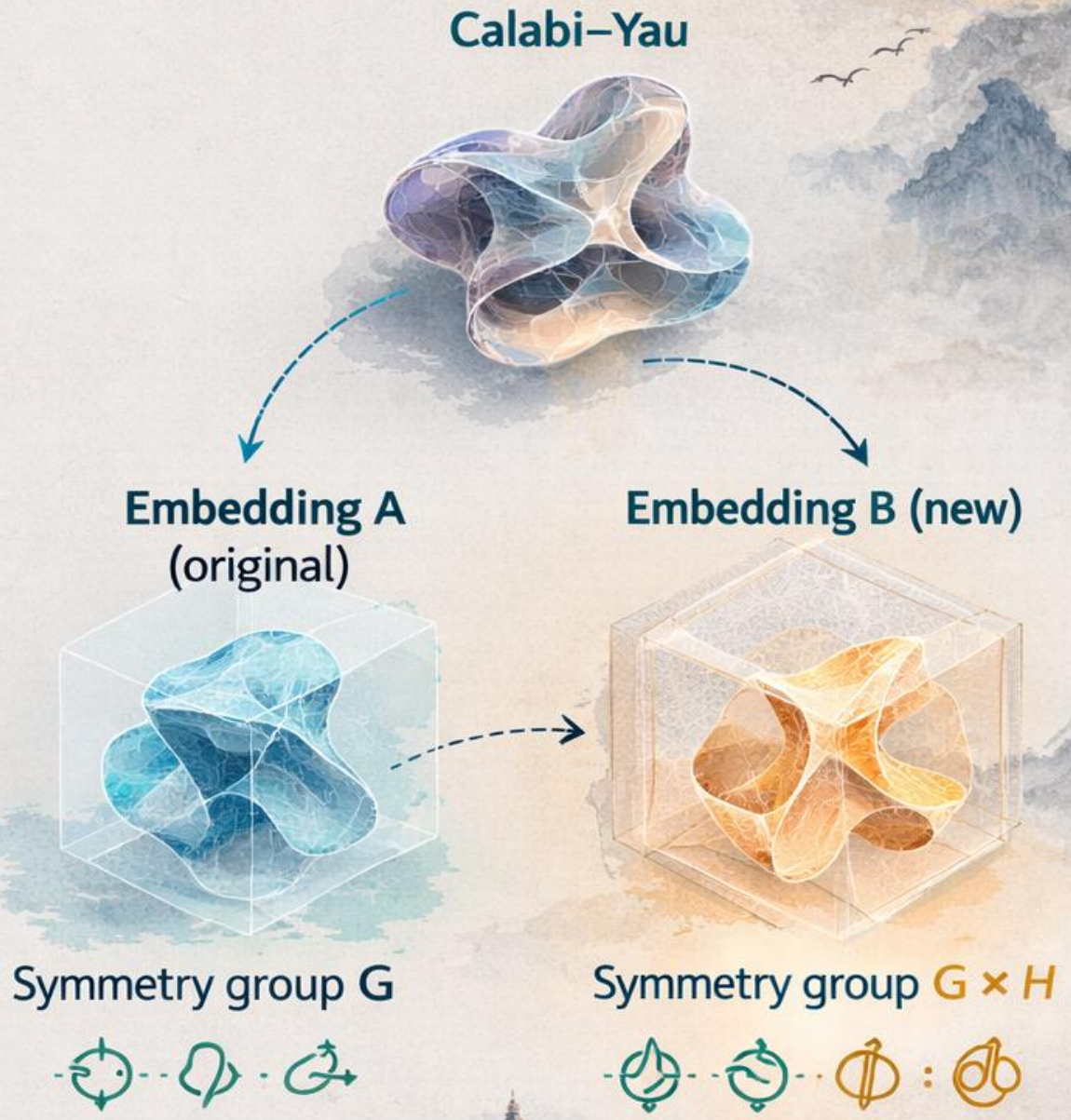
The work of Anderson, Gray, Lukas and Palti (2012) constructed models on the free quotients of CICYs in their original representations.

These free quotients were classified by Braun (2010).

But Braun's classification depends on the **embedding** of the Calabi–Yau into the ambient space, and only the **original representations** of CICYs were considered.

Question: Can there be more free quotients for model building?

Answer: **Yes!** Gray, Wang (2022)



More vocabulary brought into the dictionary in the AI era

In string theory model building, we naturally meet huge datasets of Calabi-Yau manifolds, together with huge number of rather time-consuming calculations on those Calabi-Yaus. Thus string compactification is a natural arena to apply modern AI technology and there have been many attempts in this direction.

Computing the Hodge numbers, Bull et al(2018), He and Lukas(2020), Keita(2024), He et al(2024)…

Evaluating the numerical metrics, Anderson et al(2020), Jejjala et al(2020), Douglas et al(2020), Lafors et al(2021), Perglund et al (2022)



Calculating line bundle cohomologies, Klaewer and Schlechter(2018), Brodie et al(2019)

Predicting gCICYs, Cui, Gao and Wang(2022), predicting free quotients of CICYs, Cui et al (2025)

More possibilities in the future

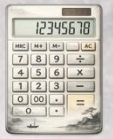
Geometry



How can generalized complete intersection Calabi–Yau (gCICY) manifolds be systematically classified?

How can the classification of freely acting discrete symmetries on toric Calabi–Yau manifolds be extended?

Computational Methods



Can modern machine learning techniques be used to efficiently approximate Ricci-flat metrics on general Calabi–Yau manifolds?

How can line bundle cohomology on Calabi–Yau threefolds be computed more efficiently, using either improved algebraic methods or machine learning approaches?

Physics



How do geometric structures of Calabi–Yau manifolds — such as topology, and discrete symmetries — shape the axion sector of four-dimensional effective theories, including their number, couplings, and mass scales?

路漫漫其修远兮，吾将上下而求索

屈原《离骚》

The road is long and stretches far away; I shall seek up and down without delay.

Qu Yuan 《Li Sao》

Thank you!