Cobordism Utopia:

Bordisms, Dualities, and the Swampland

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Based On

Utopia:
hep-th/2505.15885 w/ Braeger, Debray, Dierigl, Montero

IIBordia:
hep-th/2302.00007 w/ Debray, Dierigl, Montero

A Provocative Conjecture

McNamara Vafa ’19

$$\Omega_*^{\mathrm{QG}} = 0$$

A Provocative Conjecture

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"Cobordism Groups of Quantum Gravity are Trivial"

A Provocative Conjecture

McNamara Vafa ’19

$$\Omega_*^{\mathrm{QG}} = 0$$

“Cobordism Groups of Quantum Gravity are Trivial”

Q1: Definition of “QG”?

A Provocative Conjecture

McNamara Vafa '19

$$\Omega_*^{\mathrm{QG}} = 0$$

"Cobordism Groups of Quantum Gravity are Trivial"

Q1: Definition of "QG"?

Q2: Definition of "Ω_*^{QG}"?

A Provocative Conjecture

McNamara Vafa '19

$$\Omega_*^{\mathrm{QG}} = 0$$

"Cobordism Groups of Quantum Gravity are Trivial"

Q1: Definition of "QG"?

Q2: Definition of "Ω_*^{QG}"?

Q3: Mathematically: ill-defined, what is it good for?

A Provocative Conjecture

McNamara Vafa '19

$$\Omega_*^{\mathrm{QG}} = 0$$

"Cobordism Groups of Quantum Gravity are Trivial"

Q1: Definition of "QG"?

Q2: Definition of "Ω_*^{QG}"?

Q3: Mathematically: ill-defined, what is it good for?

A: Learn about QG / Predict new objects!

Plan of the Talk

- Symmetries, Gravity, and the Swampland
- U-Dualities
- Utopia: $\Omega_k^{\text{Spin}}(BG_U)$
- Future / Conclusions

Symmetries, Gravity, and the Swampland

“Conventional” Symmetries

Quantum (QFT_{1D}): $\mathcal{H}_{\text{Hilbert}}$ and $\widehat{H}_{\text{Hamiltonian}}$

Ray in $\mathcal{H}_{\text{Hilbert}} \Rightarrow$ State $|\psi\rangle$

Symmetry Op $\widehat{U}$: $[\widehat{U}, \widehat{H}] = 0$

Global Symmetry: $\widehat{U}|\psi\rangle = |\psi^{(U)}\rangle \neq |\psi\rangle$

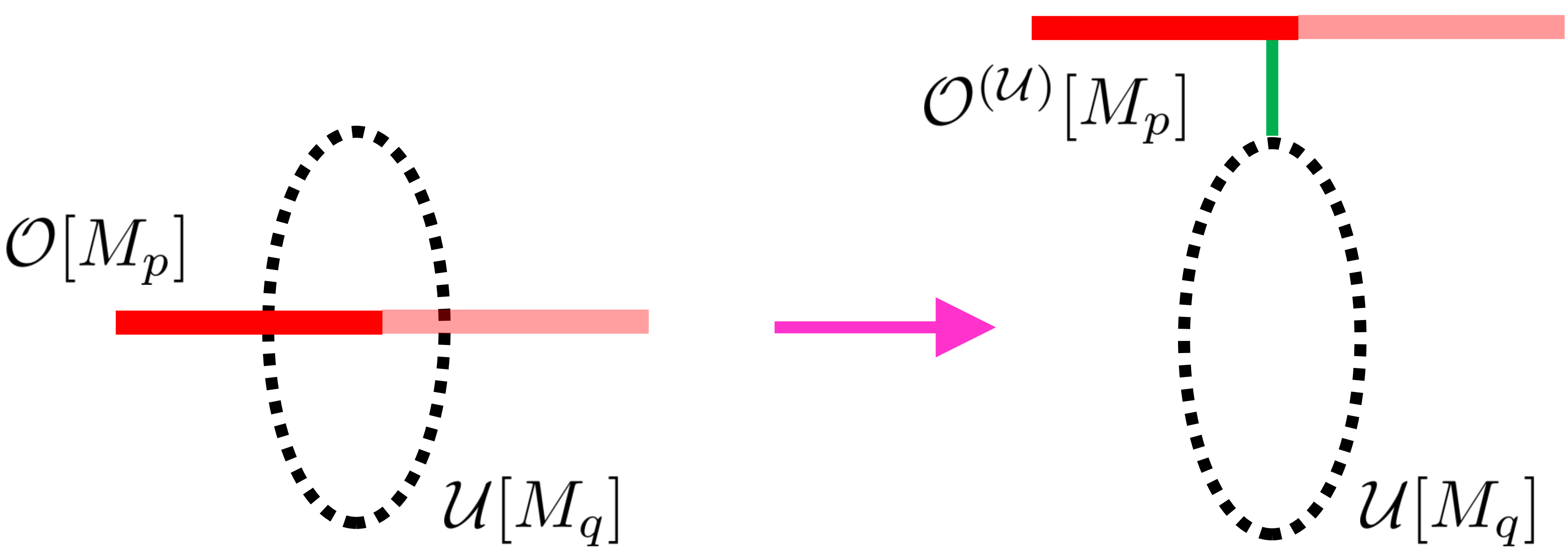
Gauge Symmetry: $\widehat{U}|\psi\rangle = |\psi^{(U)}\rangle \sim |\psi\rangle$
(Redundancy)

Generalized Symms in QFT_D

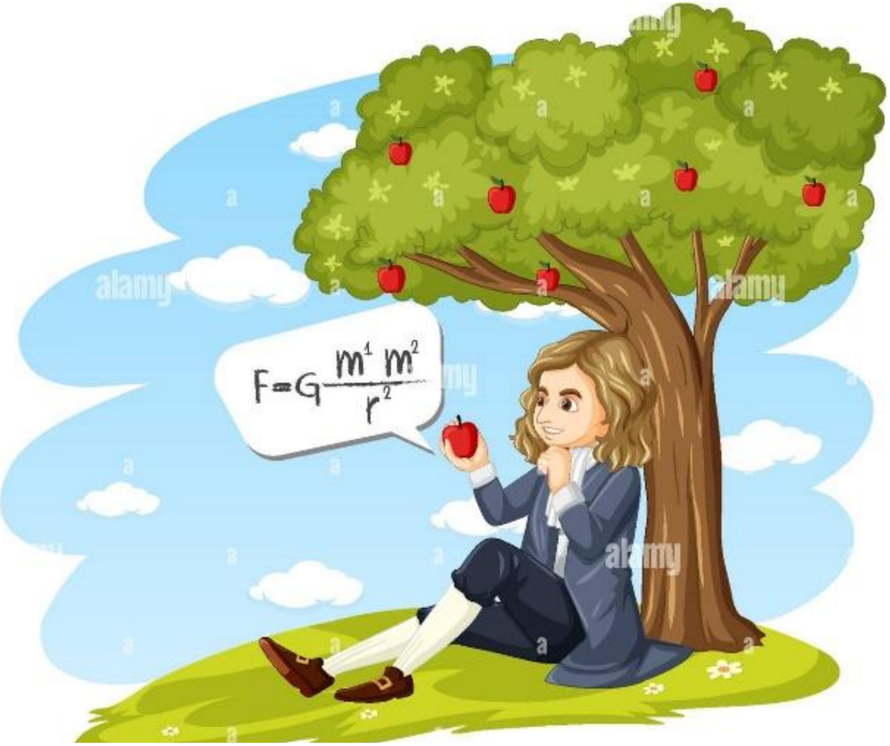
Gaiotto Kapustin Seiberg Willett '14

Global Symmetries are *Topological*

Linking / Crossing: $p + q \geq D - 1$

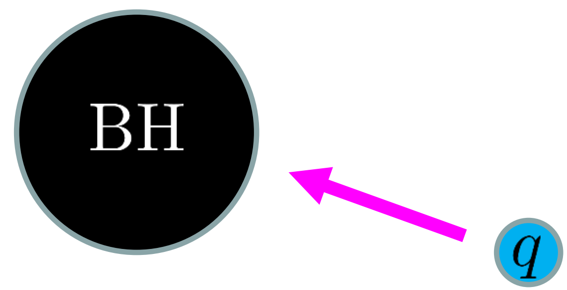


+ Gravity?



Symmetries and Gravity

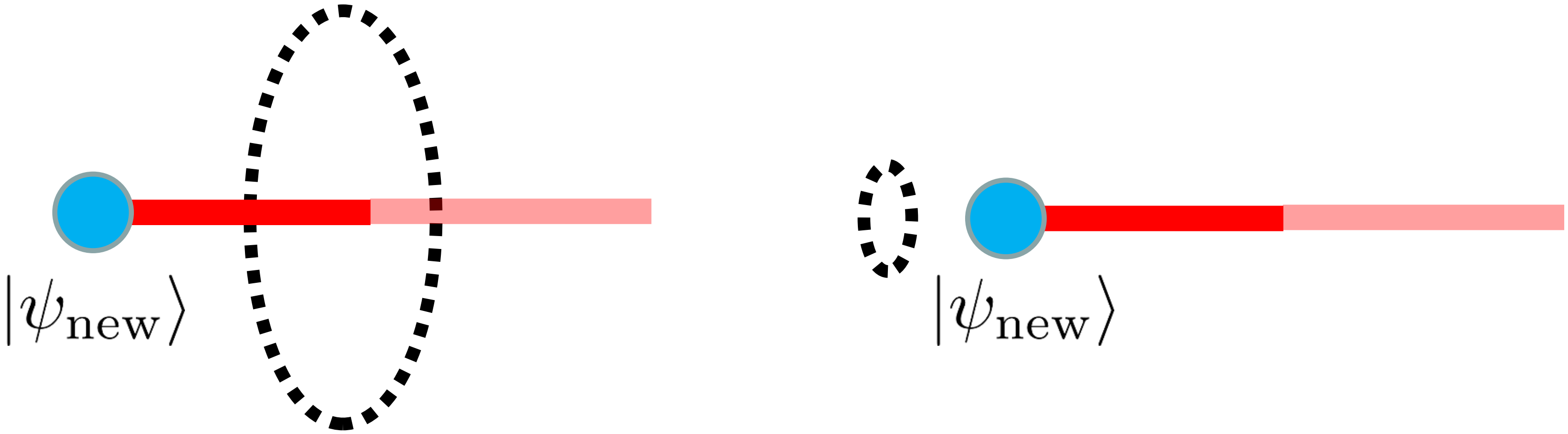
Lore: "Gauged or Broken"



Banks Dixon '88; Susskind '95; Banks Seiberg '10;
Harlow Ooguri '18; JJH Hübner Murdia '24

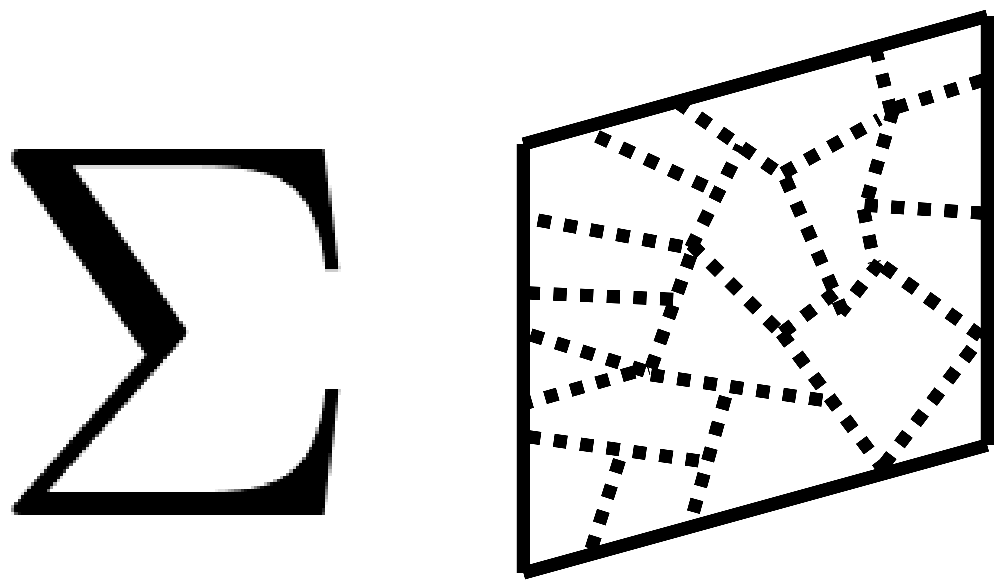
Breaking a p-form Symmetry

Introduce new states / objects:



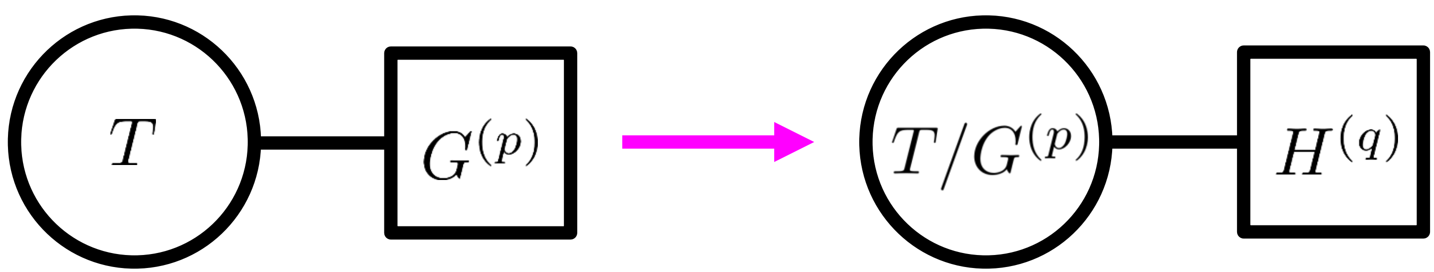
What About Gauging?

Gauging finite symmetry: sum a mesh of top ops.

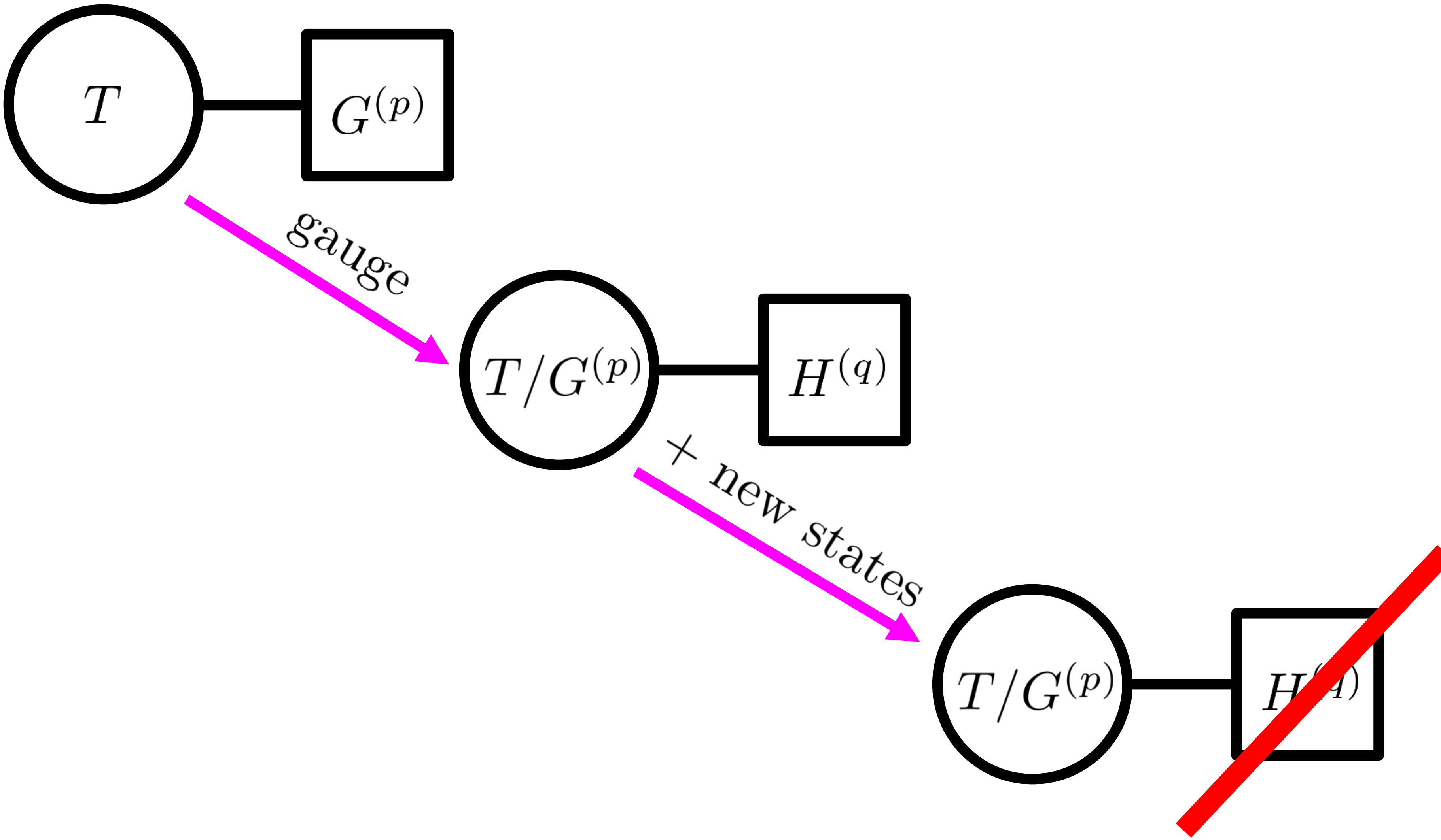


What About Gauging?

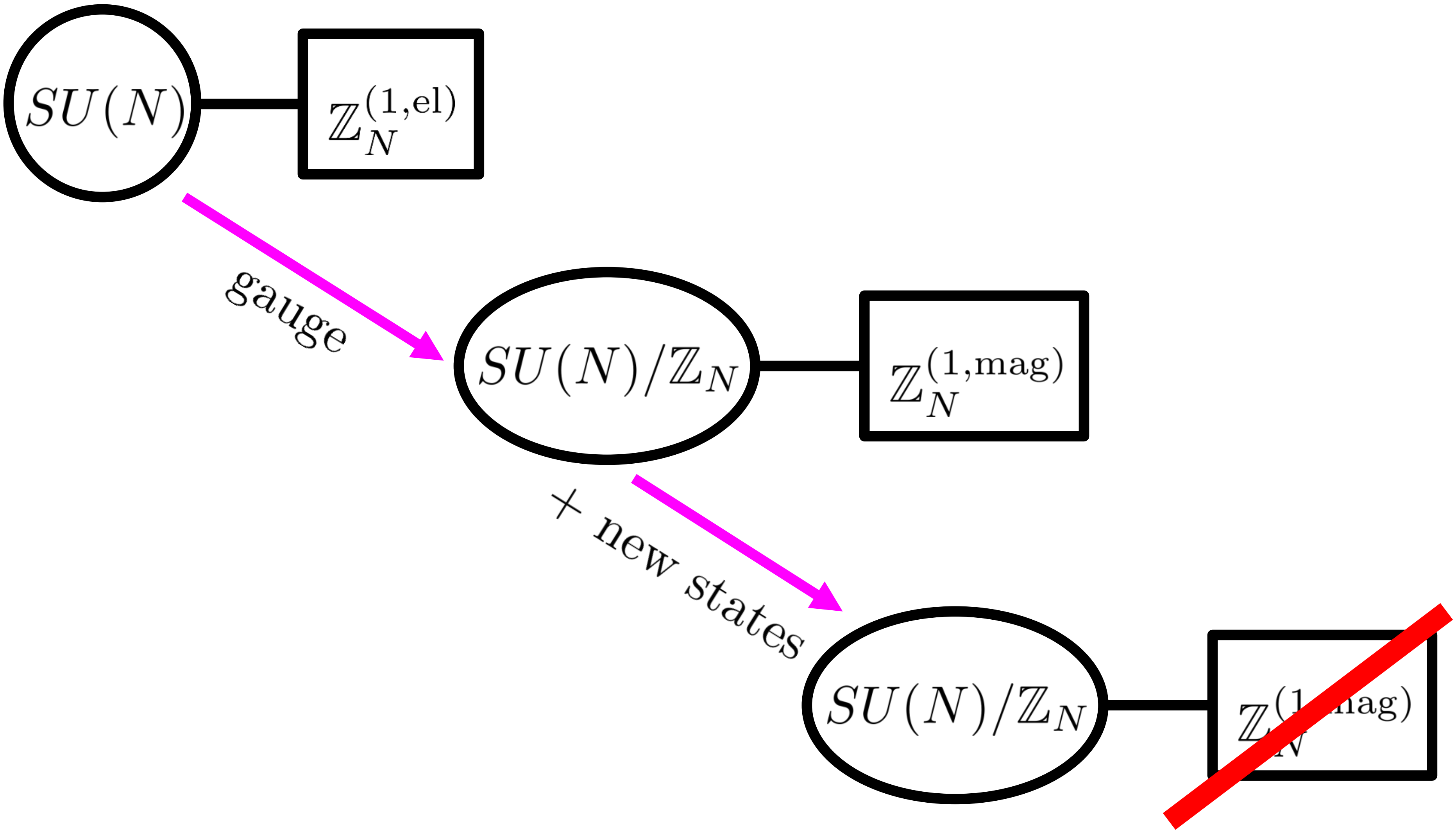
Example Issue: Gauge a finite p-form symm,
get a magnetic q-form symmetry
$(p + q = D - 2)$



Gauging + Breaking



Example

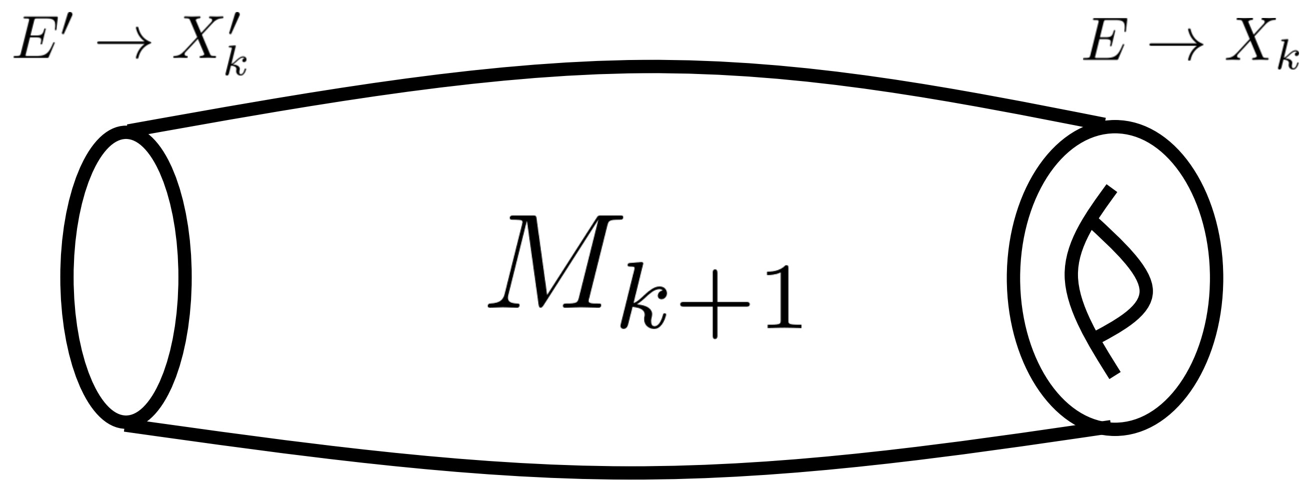


$SU(N)$
$\mathbb{Z}_N^{(1,\mathrm{el})}$
gauge
$SU(N)/\mathbb{Z}_N$
$\mathbb{Z}_N^{(1,\mathrm{mag})}$
+ new states
$SU(N)/\mathbb{Z}_N$
$\mathbb{Z}_N^{(1,\mathrm{mag})}$

¿Cobordisms & Symmetries?

Cobordism: $\Omega_k = \{\text{Equiv Classes } [X_k]\}$

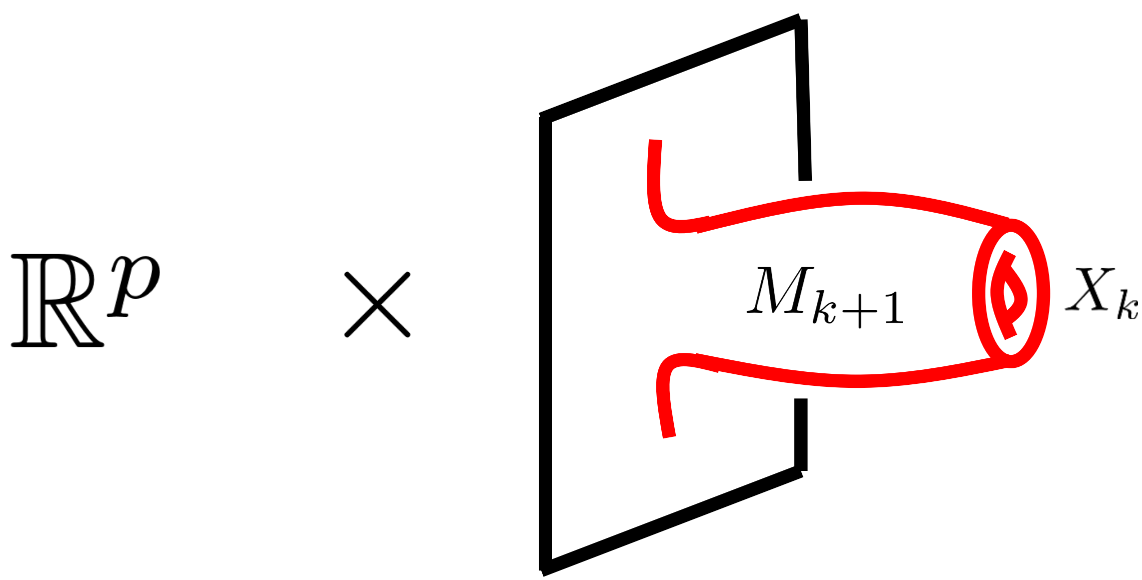
Generalized (co)homology theory



$\Omega_k \neq 0 \Rightarrow$ Global Symmetry

Consider Gravity in $D = p + (k+1)$ Dimensions

Glue in bordism defect:



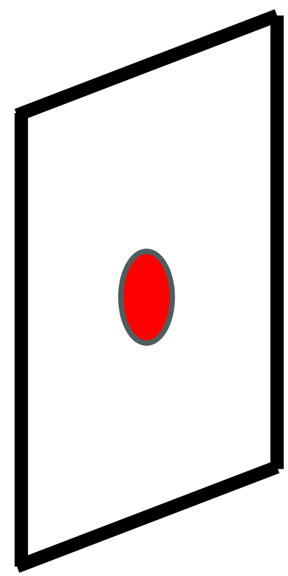
$\Omega_k \neq 0 \Rightarrow$ Global Symmetry

Consider Gravity in $D = p + (k+1)$ Dimensions

Glue in bordism defect:

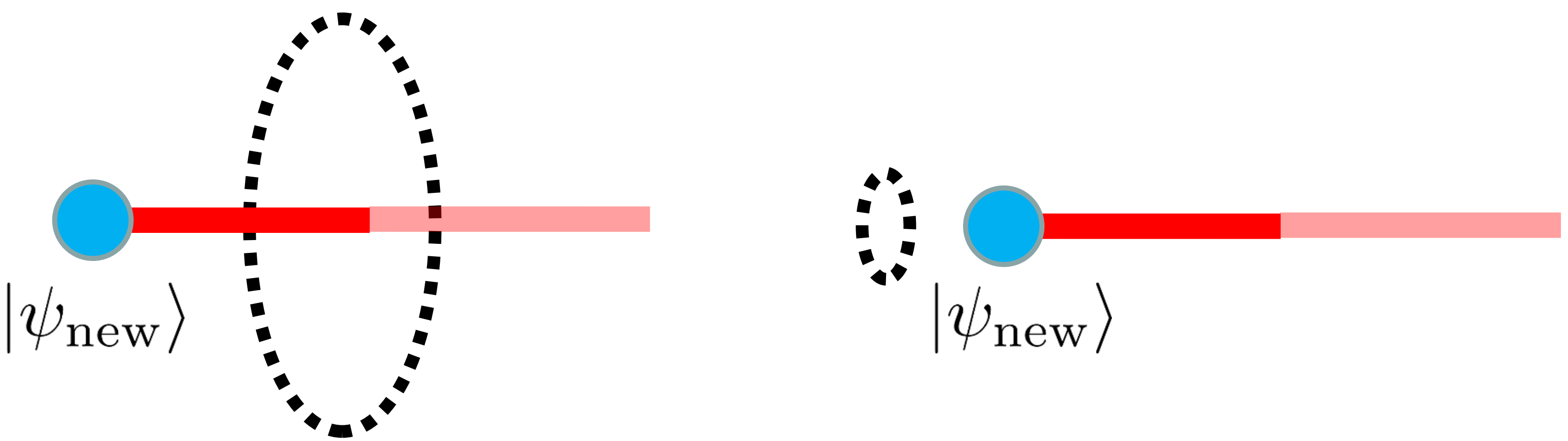
$\Rightarrow$ p-form Global Symmetry $\text{Hom}(\Omega_k, U(1))$

$\mathbb{R}^p \quad \times$



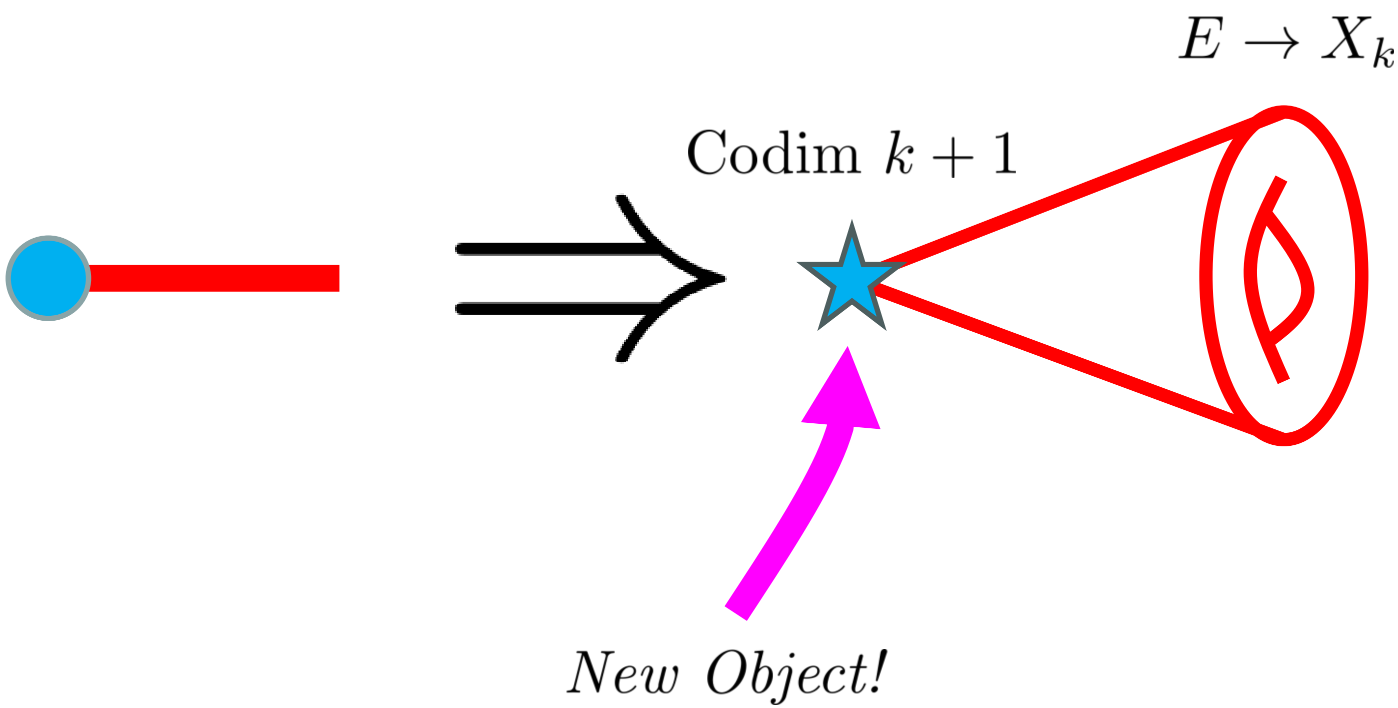
Recall:

Break a p-form symmetry with new states / objects



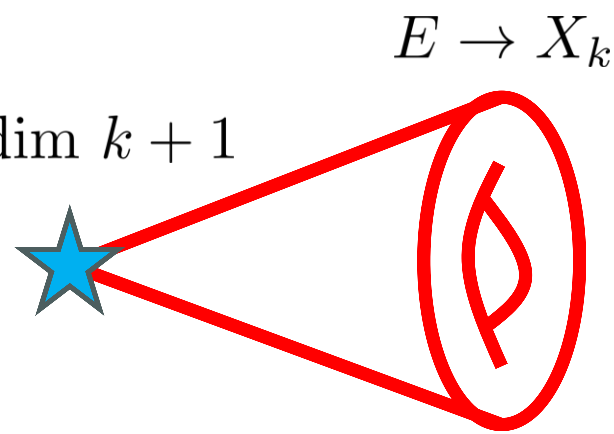
Recall:

Break a p-form symmetry with new states / objects



Implications of $\Omega_k^{\text{QG}} = 0$

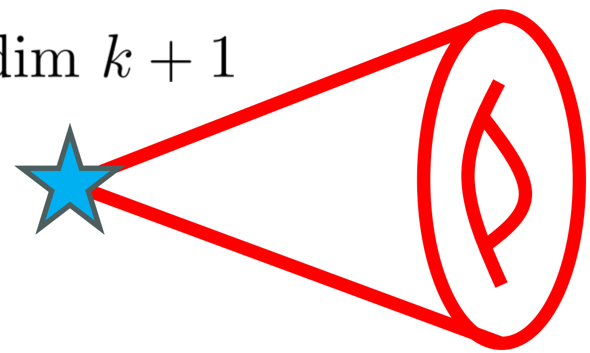
McNamara Vafa '19



Typically, $\Omega_k^{\xi}(BG) \neq 0 \Rightarrow$ Predict New Objects!

Implications of $\Omega_k^{\mathrm{QG}} = 0$

McNamara Vafa '19



tangential structure

Typically, $\Omega_k^{\xi}(BG) \neq 0 \Rightarrow$ Predict New Objects!

bundle structure group

Choosing ξ and G

Different theories have different choices

Today: $\xi = \mathsf{Spin}$ and $G =$ U-Duality Group

More Generally: ξ-twisted G structure also natural

(see IIB Dualities and Cobordisms; Debray Dierigl JJH Montero '23)

U-Dualities

Max SUSY SUGRA$_D$

(assume non-chiral)

- D-dim Gravity (Fixed Field Content)

- 32 Real Supercharges, Spinors of Spin$(D-1,1)$

- Non-Perturbative Data: States in Reps of G_U

  $G_U = \mathrm{SL}(d,\mathbb{Z}) \bowtie \mathrm{SO}(d-1,d-1;\mathbb{Z})$

  $D + d = 11$

Cremmer '80; Julia '80; Hull Townsend '94; Obers Pioline '98

U-Dualities in D-dimensions

$$G_U = \mathrm{SL}(d, \mathbb{Z}) \bowtie \mathrm{SO}(d-1, d-1; \mathbb{Z})$$

U-Dualities in D-dimensions

Consider M-theory on $\mathbb{R}^{D-1,1} \times T^d$

$$G_U = \mathrm{SL}(d, \mathbb{Z}) \bowtie \mathrm{SO}(d-1, d-1; \mathbb{Z})$$

U-Dualities in D-dimensions

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Large Diffs on T^d

U-Dualities in D-dimensions

Consider M-theory on $\mathbb{R}^{D-1,1} \times T^d$

$$G_U = \mathrm{SL}(d, \mathbb{Z}) \bowtie \mathrm{SO}(d-1, d-1; \mathbb{Z})$$

Large Diffs on T^d

T-dualities on T^{d-1}
(see mirror symm)

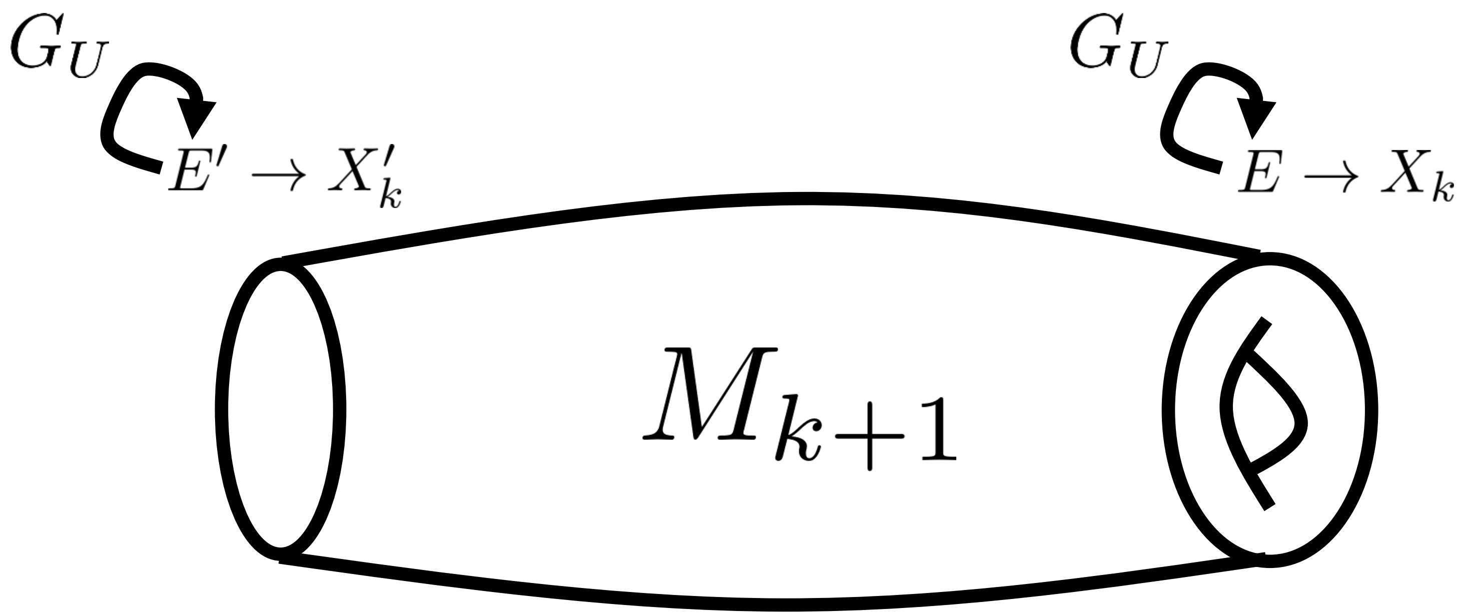
Cremmer '80; Julia '80; Hull Townsend '94

U-Duality Groups

D	G_U
10	1
9	$\mathrm{SL}(2,\mathbb{Z})$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$
7	$\mathrm{SL}(5,\mathbb{Z})$
6	$\mathrm{SO}(5,5,\mathbb{Z})$
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$

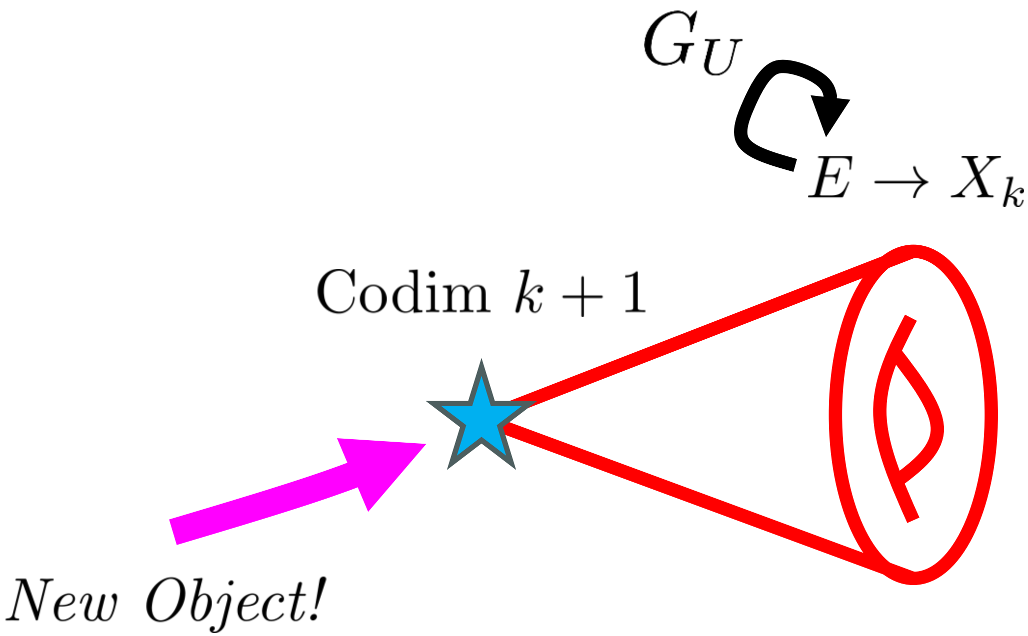
Cobordisms & Dualities

Cobordism Groups $\Omega_k^{\mathrm{Spin}}(BG_U)$



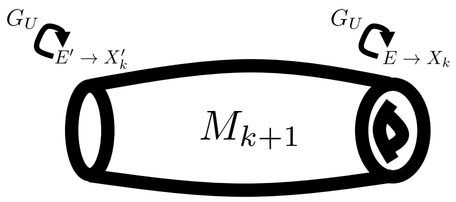
Cobordisms & Swampland

Cobordism Groups $\Omega_k^{\mathrm{Spin}}(BG_U)$ $(1 \leq k \leq D-1)$



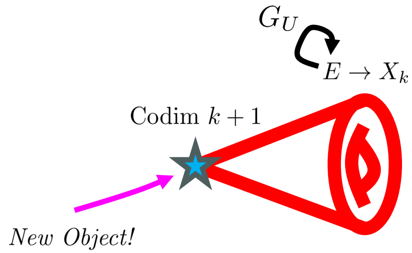
The Plan

- Compute $\Omega_k^{\text{Spin}}(BG_U) \simeq \mathbb{Z}^{\oplus m} \oplus \bigoplus_i \mathbb{Z}/p_i^{\ell_i}\mathbb{Z}$



- Find representatives $[X_k] \in \Omega_k^{\text{Spin}}(BG_U)$

- Find String Backgrounds: $\text{Cone}(X_k) = Y_{k+1}$



Utopia: $\Omega_1(BG_U)$

Braeger Debray Dierigl JJH Montero '25

U-Duality Groups

D	G_U
10	1
9	$\mathrm{SL}(2,\mathbb{Z})$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$
7	$\mathrm{SL}(5,\mathbb{Z})$
6	$\mathrm{SO}(5,5,\mathbb{Z})$
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$

Computing $\Omega_1^{\text{Spin}}(BG_U)$

Atiyah Hirzebruch Spectral Sequence

$$E^2_{p,q} = H_p(BG_U, \Omega_q^{\text{Spin}}(pt))$$

$$d_2 : E^2_{p,q} \to E^2_{p-2,q+1}$$

$$\Rightarrow \Omega_1^{\text{Spin}}(BG_U) = H_1(BG_U) \oplus \Omega_1^{\text{Spin}}(pt)$$

$\text{Ab}(G_U)$ $\qquad$ $\mathbb{Z}_2$

Abelianizations

D	G_U	$\mathrm{Ab}(G_U)$
10	1	1
9	$\mathrm{SL}(2,\mathbb{Z})$	$\mathbb{Z}_3 \oplus \mathbb{Z}_4$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$	$\mathbb{Z}_3 \oplus \mathbb{Z}_4$
7	$\mathrm{SL}(5,\mathbb{Z})$	1
6	$\mathrm{SO}(5,5,\mathbb{Z})$	1
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$	1
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$	1
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$	1

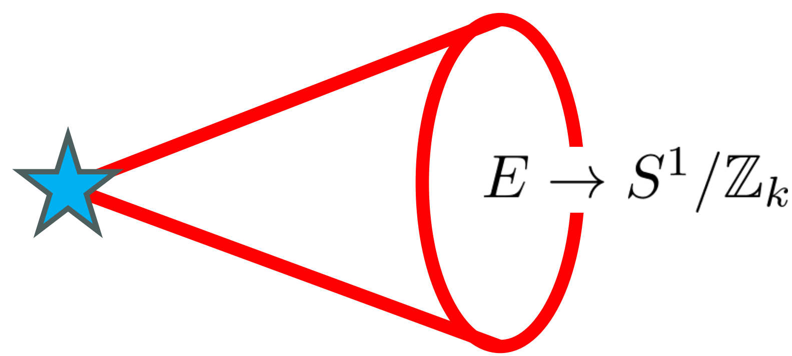
Geometrization

$$G_U = \mathrm{SL}(2,\mathbb{Z}) \Rightarrow \mathrm{Ab}(G_U) = \mathbb{Z}_3 \oplus \mathbb{Z}_4$$

M-theory Background: $T^2 \to 9D$ spacetime

$\mathbb{Z}_3 : \partial(\mathbb{C} \times T^2/\mathbb{Z}_3)$

$\mathbb{Z}_4 : \partial(\mathbb{C} \times T^2/\mathbb{Z}_4)$



(similar structure in 8D F-theory bkgnds)

Utopia: $\Omega_k(BG_U^{8\mathrm{D}})$

Braeger Debray Dierigl JJH Montero '25

U-Duality Groups

D	G_U	$\mathrm{Ab}(G_U)$
10	1	1
9	$\mathrm{SL}(2,\mathbb{Z})$	$\mathbb{Z}_3 \oplus \mathbb{Z}_4$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$	$\mathbb{Z}_3 \oplus \mathbb{Z}_4$
7	$\mathrm{SL}(5,\mathbb{Z})$	1
6	$\mathrm{SO}(5,5,\mathbb{Z})$	1
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$	1
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$	1
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$	1

U-Duality Groups

D	G_U	$\mathrm{Ab}(G_U)$
10	1	1
9	$\mathrm{SL}(2,\mathbb{Z})$	$\mathbb{Z}_3 \oplus \mathbb{Z}_4$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$	$\mathbb{Z}_3 \oplus \mathbb{Z}_4$
7	$\mathrm{SL}(5,\mathbb{Z})$	1
6	$\mathrm{SO}(5,5,\mathbb{Z})$	1
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$	1
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$	1
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$	1

Why $D = 8$?

$G_U^{8D} = \mathrm{SL}(3, \mathbb{Z}) \times \mathrm{SL}(2, \mathbb{Z})$

More is known about group cohomology

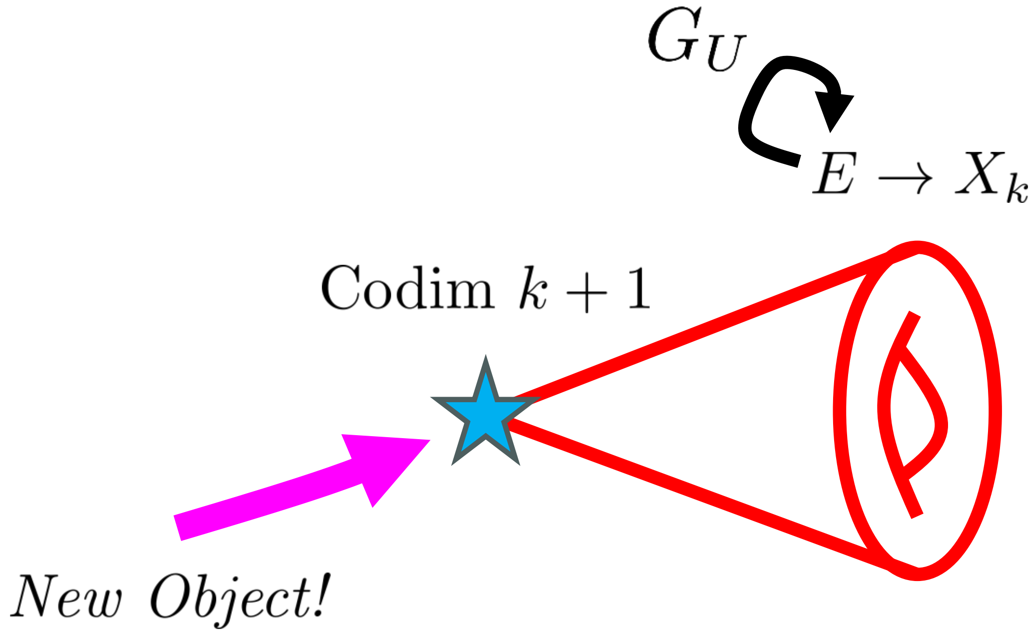
Soulé '78, Minami '94

$\mathrm{SL}(2, \mathbb{Z})$ & Spin / Pin^+ lifts already done

"IIBordia" Debray Dierigl JJH Montero '23

Cobordisms & Swampland

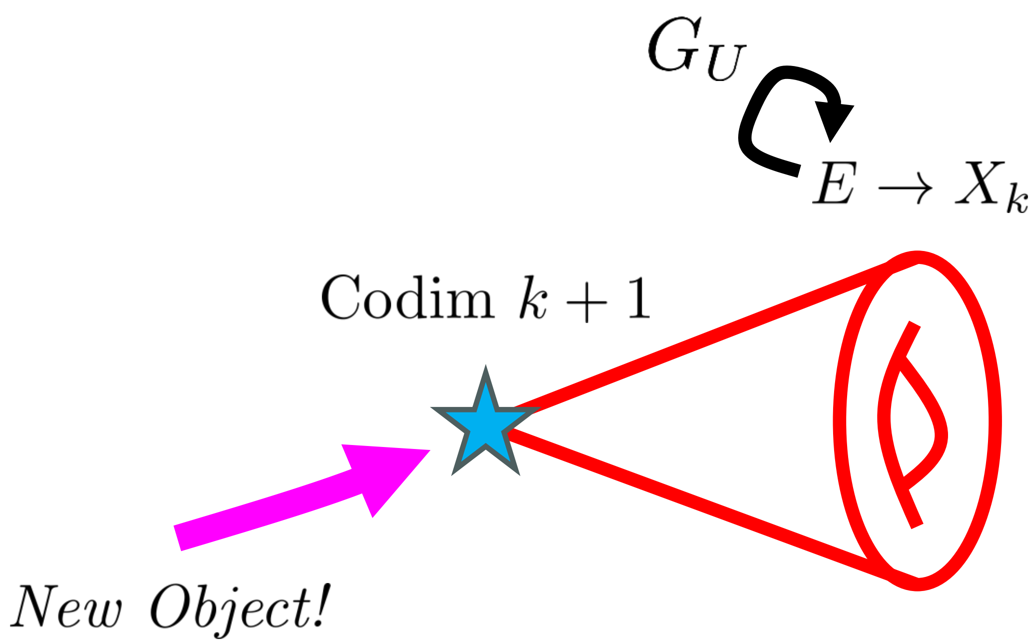
Cobordism Groups $\Omega_k^{\text{Spin}}(BG_U)$ $(1 \leq k \leq D-1)$



$D = 8$ U-Dualities

$$G_U = \mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$$

Cobordism Groups $\Omega_k^{\mathrm{Spin}}(BG_U)$ $(1 \leq k \leq 7)$



Bordism Groups

Localization at primes $p = 2, 3$

k	$\Omega_k^{\mathrm{Spin}}\big(BSL(2,\mathbb{Z}) \times BSL(3,\mathbb{Z})\big)$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4$
2	$\mathbb{Z}_2^{\oplus 4}$
3	$\mathbb{Z}_3^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_8^{\oplus 3}$
4	$\mathbb{Z} \oplus \mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
5	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_2^{\oplus 5} \oplus \mathbb{Z}_4$
6	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
7	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 6} \oplus \mathbb{Z}_8^{\oplus 2} \oplus \mathbb{Z}_{16}^{\oplus 2} \oplus \mathbb{Z}_{32}$

Bordism Groups

Localization [illegible] primes $p = 2, 3$

¿Explicit Generators?
¿Explicit Generators?
¿Explicit Generators?
¿Explicit Generators?

k	Ω^{Spin}_k [illegible] $\mathbb{Z}) \times$ [illegible] $3, \mathbb{Z}))$
[illegible]	$\mathbb{Z}_2 \oplus$ [illegible] $\mathbb{Z}_4$
2	$\mathbb{Z}_2^{\oplus}$ [illegible]
3	$\mathbb{Z}_3^{\oplus}$ [illegible] $\oplus \mathbb{Z}_8$
4	[illegible] $^{\oplus 2} \oplus$ [illegible] $\mathbb{Z}_4^{\oplus 2}$
5	$\mathbb{Z}_3^{\oplus}$ [illegible] $\oplus \mathbb{Z}_2^{\oplus}$ [illegible]
6	$\mathbb{Z}_3^{\oplus 2} \oplus$ [illegible] $\mathbb{Z}_4^{\oplus 2}$
7	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 6} \oplus$ [illegible] $\mathbb{Z}_{16}^{\oplus 2} \oplus \mathbb{Z}_{32}$

Geometrizing Dualities

$$G_U = \mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$$

Geometrizing $\mathrm{SL}(3,\mathbb{Z})$

$$G_U = \boxed{\mathrm{SL}(3,\mathbb{Z})} \times \mathrm{SL}(2,\mathbb{Z})$$

M-th on $(T^3 \to \mathrm{Cone}(X_k)) \Rightarrow$

$$\mathrm{SL}(3,\mathbb{Z}) \circlearrowleft T^3$$

$$\begin{array}{c} T^3 \\ \downarrow \\ \mathrm{Cone}(X_k) \end{array}$$

Geometrizing $\mathrm{SL}(2,\mathbb{Z})$ (1 / 2)

$$G_U = \mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$$

$$\mathrm{SL}(3,\mathbb{Z}) \cup \mathrm{SL}(2,\mathbb{Z})$$

$$\mathrm{SL}(2,\mathbb{Z})$$

$$\text{M-th on } (S^1 \times T^2 \to \mathrm{Cone}(X_k)) \Rightarrow \quad S^1 \times \begin{array}{c} T^2 \\ \downarrow \\ \mathrm{Cone}(X_k) \end{array}$$

Geometrizing $\mathrm{SL}(2,\mathbb{Z})$ (1.5 / 2)

$$\begin{array}{ccc} G_U = & \mathrm{SL}(3,\mathbb{Z}) & \times\ \mathrm{SL}(2,\mathbb{Z}) \\ & \cup & \\ & \mathrm{SL}(2,\mathbb{Z}) & \end{array}$$

$$\text{F-th on } (T_F^2 \times T^2 \to \mathrm{Cone}(X_k)) \Rightarrow \quad T^2 \quad \times \begin{array}{c} \mathrm{SL}(2,\mathbb{Z}) \\ \curvearrowleft \\ T_F^2 \\ \downarrow \\ \mathrm{Cone}(X_k) \end{array}$$

Geometrizing $\mathrm{SL}(2, \mathbb{Z})$ (2 / 2)

$$G_U = \mathrm{SL}(3, \mathbb{Z}) \times \boxed{\mathrm{SL}(2, \mathbb{Z})}$$

$$\mathrm{SL}(2, \mathbb{Z})$$

$$\text{F-th on } (T^2_F \to T^2 \times \mathrm{Cone}(X_k)) \Rightarrow \begin{array}{c} T^2 \\ \downarrow \\ T^2_F \times \mathrm{Cone}(X_k) \end{array}$$

k	$\Omega_k^{\mathrm{Spin}}(B\mathrm{SL}(2,\mathbb{Z}) \times B\mathrm{SL}(3,\mathbb{Z}))$	Generators	Defect
1	$\mathbb{Z}_2$	S^1_+	spin defect
	$\mathbb{Z}_3$	$S^1_{\gamma_3}$	$\mathcal{N} = (2,0)$ 6d SCFTs
	$\mathbb{Z}_4$	$S^1_{\gamma_4}$	$\mathcal{N} = (2,0)$ 6d SCFTs
2	$\mathbb{Z}_2$	$S^1_+ \times S^1_+$	spin defect on S^1
	$\mathbb{Z}_2$	$S^1_+ \times S^1_{\gamma_4}$	codimension-two defect on S^1_+
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$S^1_{M_1^{(i)}} \times S^1_{M_2^{(i)}}$	twisted compactification of additional codimension-two objects (hosting SCFTs)
3	$\mathbb{Z}_3$	L^3_{3,γ_3}	type IIB on singular local Calabi-Yau
	$\mathbb{Z}_3$	$L^3_{3,\Gamma_3^{(1)}}$	non-Higgsable cluster on T^2
	$\mathbb{Z}_3$	$L^3_{3,\Gamma_3^{(2)}}$	twisted compactification of 5d SCFTs
	$\mathbb{Z}_2$	$S^1_+ \times S^1_+ \times S^1_{\gamma_4}$	codimension-two defect on $S^1_+ \times S^1_+$
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$S^1_{\gamma_4} \times S^1_{M_1^{(i)}} \times S^1_{M_2^{(i)}}$	(non-geometrically) twisted compactification of codimension-two defect
	$\mathbb{Z}_8$	L^3_{4,γ_4}	type IIB on singular geometry
	$\mathbb{Z}_8$	$L^3_{4,\Gamma_4^{(1)}}$	F-theory on singular geometry
	$\mathbb{Z}_8$	$L^3_{4,\Gamma_4^{(2)}}$	M-theory on singular geometry
4	$\mathbb{Z}$	K3	codimension-five spin defect
	$\mathbb{Z}_3 \oplus \mathbb{Z}_3$	$S^1_{\gamma_3} \times L^3_{3,\Gamma_3^{(i)}}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_4 \oplus \mathbb{Z}_4$	$S^1_{\gamma_4} \times L^3_{4,\Gamma_4^{(i)}}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_2$	W_4	(topolog.) twisted compactification of defect
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$A\,,\,A'$	(non-geom.) twisted compactification of defects

k	$\Omega_k^{\text{Spin}}(BSL(2,\mathbb{Z}) \times BSL(3,\mathbb{Z}))$	Generators	Defect
5	$\mathbb{Z}_9$	L^5_{3,γ_3}	type IIB on singular local Calabi-Yau (CY)
	$\mathbb{Z}_3$	$L^5_{3,(\gamma_3,\Gamma_3^{(1)})}$	composite S-string (see [28])
	$\mathbb{Z}_3$	$L^5_{3,(\gamma_3,\Gamma_3^{(2)})}$	non-geometric string
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$S^1_{\gamma_4} \times A\,,\, S^1_{\gamma_4} \times A'$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_2$	$S^1_{\gamma_4} \times W_4$	(non-geometrically and topological) twisted compactification of defect
	$\mathbb{Z}_4$	Q^5_{4,γ_4}	(topolog.) twisted compactification of defect
	$\mathbb{Z}_2$	$Q^5_{4,(\gamma_4,\Gamma_4^{(1)})}$	composite S-fold compactification with topological twist
	$\mathbb{Z}_2$	$Q^5_{(\gamma_4,\Gamma_4^{(2)})}$	non-geometric 3-branes compactified with topological twist
6	$\mathbb{Z}_3 \oplus \mathbb{Z}_3$	$L^3_{3,\gamma_3} \times L^3_{3,\Gamma_3^{(i)}}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_4 \oplus \mathbb{Z}_4$	$L^3_{4,\gamma_4} \times L^3_{4,\Gamma_4^{(i)}}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{RP}^3_{M_1^{(i)}} \times \mathbb{RP}^3_{M_2^{(i)}}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_2$	W_6	(non-geom.) twisted compactification of additional codimension-four S-fold
7	$\mathbb{Z}_9$	L^7_{3,γ_3}	type IIB on singular local CY
	$\mathbb{Z}_9$	$L^7_{3,\Gamma_3^{(1)}}$	composite S-string on T^2 (see [28])
	$\mathbb{Z}_9$	$L^7_{3,\Gamma_3^{(2)}}$	twisted M-theory compact. on singular local CY
	$\mathbb{Z}_3$	$L^7_{3,(\gamma_3,\Gamma_3^{(1)})}$	composite S-instanton (see [28])
	$\mathbb{Z}_3$	$L^7_{3,(\gamma_3,\Gamma_3^{(2)})}$	non-geometric instanton
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$(L^3_{4,\Gamma_4^{(i)}} \times \text{K3})/\mathbb{Z}_2$	(topolog.) twisted compactification of defects
	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$S^1_{\gamma_4} \times \mathbb{RP}^3_{M_1^{(i)}} \times \mathbb{RP}^3_{M_2^{(i)}}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_2$	$S^1_{\gamma_4} \times W_6$	(non-geom.) twisted compactification of defect
	$\mathbb{Z}_2 \oplus \mathbb{Z}_{32}$	$L^7_{4,\gamma_4}\,,\, \widetilde{L}^7_{4,\gamma_4}$	type IIB on singular geometry
	$\mathbb{Z}_8 \oplus \mathbb{Z}_8$	$(T^4 \times \mathbb{RP}^3)_{(\gamma_4,M_1^{(i)},M_2^{(i)})}$	(non-geom.) twisted compactification of defects
	$\mathbb{Z}_{16} \oplus \mathbb{Z}_{16}$	$L^7_{4,\Gamma_4^{(i)}}$	F-/M-theory on singular geometry

Geometric Examples

$\Omega_3^{\text{Spin}}(BG_U)$	$[X_3]$	String / M- / F-theory Bkgnd
$\mathbb{Z}_3$	L^3_{3,γ_3}	type IIB on singular local Calabi-Yau
$\mathbb{Z}_3$	$L^3_{3,\Gamma_3^{(1)}}$	non-Higgsable cluster on T^2
$\mathbb{Z}_3$	$L^3_{3,\Gamma_3^{(2)}}$	twisted compactification of 5d SCFTs

Geometric Examples

Order 3 elements of $\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$

$$\gamma_3 = \begin{bmatrix} -1 & -1 \\ +1 & 0 \end{bmatrix}$$

$$\Gamma_3^{(1)} = \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & +1 & 0 \end{bmatrix}$$

$$\Gamma_3^{(2)} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & +1 \\ +1 & 0 & 0 \end{bmatrix}$$

Geometric Examples

Order 3 elements of $\mathrm{SL}(3, \mathbb{Z}) \times \mathrm{SL}(2, \mathbb{Z})$

$$\gamma_3 = \begin{bmatrix} -1 & -1 \\ +1 & 0 \end{bmatrix} \Rightarrow \text{F-theory on } T^2_F \times (T^2 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\gamma_3}$$

Geometric Examples

Order 3 elements of $\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$

$$\Gamma_3^{(1)} = \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & +1 & 0 \end{bmatrix} \Rightarrow \text{F-theory on } T^2 \times (T_F^2 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\Gamma_3^{(1)}}$$

Geometric Examples

Order 3 elements of $\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$

$$\Gamma_3^{(2)} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & +1 \\ +1 & 0 & 0 \end{bmatrix} \Rightarrow \text{M-theory on } (T^3 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\Gamma_3^{(2)}}$$

Geometric Examples

Order 3 elements of $\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$

$$\gamma_3 = \begin{bmatrix} -1 & -1 \\ +1 & 0 \end{bmatrix} \Rightarrow \text{F-theory on } T^2_F \times (T^2 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\gamma_3}$$

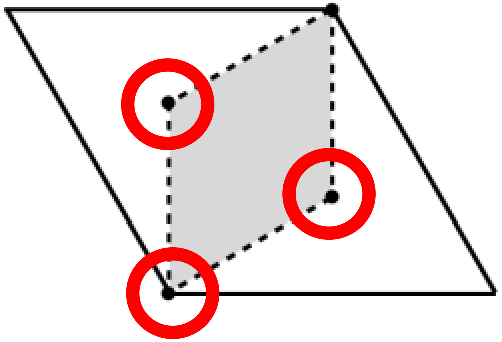
$$\Gamma_3^{(1)} = \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & +1 & 0 \end{bmatrix} \Rightarrow \text{F-theory on } T^2 \times (T^2_F \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\Gamma_3^{(1)}}$$

$$\Gamma_3^{(2)} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & +1 \\ +1 & 0 & 0 \end{bmatrix} \Rightarrow \text{M-theory on } (T^3 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\Gamma_3^{(2)}}$$

Example: γ_3

$$\gamma_3 = \begin{bmatrix} -1 & -1 \\ +1 & 0 \end{bmatrix} \Rightarrow \text{F-theory on } T^2_F \times (T^2 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\gamma_3}$$

Locally: IIB on $\mathbb{C}^3/\mathbb{Z}_3$ (3 glued copies)

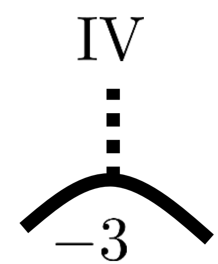


$\Rightarrow$ 4D $\mathcal{N} = 2$ Theory (but a bit "boring")

Example: $\Gamma_3^{(1)}$

$$\Gamma_3^{(1)} = \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & +1 & 0 \end{bmatrix} \Rightarrow \text{F-theory on } T^2 \times (T_F^2 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\Gamma_3^{(1)}}$$

Blowup: Local Elliptically fibered $\text{CY}_3 \times T^2$



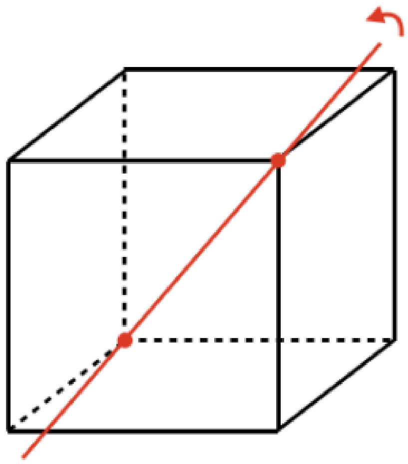
$\times T^2$

6D Superconformal Field Theory (SCFT) on a T^2

Morrison Taylor '12; JJH Morrison Vafa '13; JJH Morrison Rudelius Vafa '15

Example: $\Gamma_3^{(2)}$

$$\Gamma_3^{(2)} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & +1 \\ +1 & 0 & 0 \end{bmatrix} \Rightarrow \text{M-theory on } (T^3 \times \mathbb{C}^2)/(\mathbb{Z}_3)_{\Gamma_3^{(2)}}$$



Locally: $\mathbb{C}^3/\mathbb{Z}_3$ fibered over S^1

5D SCFT compactified on S^1 + twisting

Seiberg '96; Morrison Seiberg (Intriligator) '97

Braeger Debray Dierigl JJH Montero '25

Non-Geometric Example

$\Omega_6^{\text{Spin}}(BG_U)$	$[X_6]$	String / M- / F-theory Bkgnd
$\mathbb{Z}_3 \oplus \mathbb{Z}_3$	$L^3_{3,\gamma_3} \times L^3_{3,\Gamma_3^{(i)}}$	(non-geom.) twisted compactification of defects

Non-Geom: $[X_6] = S^3_{\gamma_3} \times S^3_{\Gamma_3^{(i)}}$

Particles (codim 7)

$$\gamma_3 = \begin{bmatrix} -1 & -1 \\ +1 & 0 \end{bmatrix} \qquad \Gamma_3^{(1)} = \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & +1 & 0 \end{bmatrix} \qquad \Gamma_3^{(2)} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & +1 \\ +1 & 0 & 0 \end{bmatrix}$$

$$\partial(\mathrm{M-th}) = X_6^{(i)} = (T^2_{\mathrm{non}} \otimes T^3_M \times S^3 \times S^3)/(\mathbb{Z}_3)_{\gamma_3, \Gamma_3^{(i)}}$$

Example of exceptional field theory bkgnd Hull '04 + ...

¿Supersymmetry?

Omissions:
Computing $\Omega_k^{\mathrm{Spin}}(BG_U^{8D})$

Brief Sketch

Strategy: Localize at primes $p = 2, 3$ (stable splitting)

$B\mathbb{Z}_4 \to B\mathrm{SL}(2, \mathbb{Z})$

2-local: $B\mathrm{SL}(3, \mathbb{F}_2) \bigvee B\mathrm{SL}(3, \mathbb{F}_2) \bigvee L(2) \to B\mathrm{SL}(3, \mathbb{Z})$

(Kunneth + Adams)

$B\mathbb{Z}_3 \to B\mathrm{SL}(2, \mathbb{Z})$

3-local: $BD_6 \bigvee BD_6 \to B\mathrm{SL}(3, \mathbb{Z})$

(Kunneth + Atiyah-Hirzebruch)

Summary / Future

What Was This Talk About?

- $\Omega_*^{\mathrm{QG}} = 0 \Rightarrow$ New Objects

- $\Omega_1^{\mathrm{Spin}}(BG_U)$ and $\Omega_k^{\mathrm{Spin}}(BG_U^{8D})$

- Spin / Pin Lifts of G_U and $\Omega_k^{\xi - G}$?

- SUSY vs Non-SUSY bkgnds?

- New Objects (e.g., R7-brane of type IIB)?