HOLOGRAPHY AND KKLT

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TWO SWAMPLAND CONJECTURES

No scale separated AdS vacua

[Gautason, Schillo, Van Riet, Williams ’15]

[Gautason, Van Hemelryck, Van Riet ’18]

[D. Lüst, Palti, Vafa ’19]

No (long-lived) de Sitter vacua

[Obied, Ooguri, Spodyneiko, Vafa ’18]

[Bedroya, Vafa ’19]

Potential counterexample:

KKLT ?

[Kachru, Kallosh, Linde, Trivedi ’03]

CHALLENGES FOR STRING PHENOMENOLOGY

Issues from compactification:

1. Deformations of compactification space

$\rightarrow$ lots of massless scalar fields (moduli)

not observed + destabilize compactification!

2. Natural: AdS or Minkowski ($\Lambda \leq 0$)

de Sitter vacua ($\Lambda > 0$): extremely difficult

(Conjecture: no (long lived) de Sitter in Quantum Gravity [Vafa et al. '18, '19])

KKLT DE SITTER VACUA

Three step procedure [KKLT '03]

1. warped IIB with CS-moduli stabilized by three-form fluxes
   + region with strong warping [GKP]
   described by Klebanov-Strassler throat [Klebanov, Strassler '00]
   → large hierarchy of scales

2. Stabilize Kähler moduli by non-perturbative effets
   → supersymmetric AdS-vacuum

3. Supersymmetry breaking by an anti-D3-brane at the bottom of the throat
   → exp. suppressed uplift to dS due to strong warping

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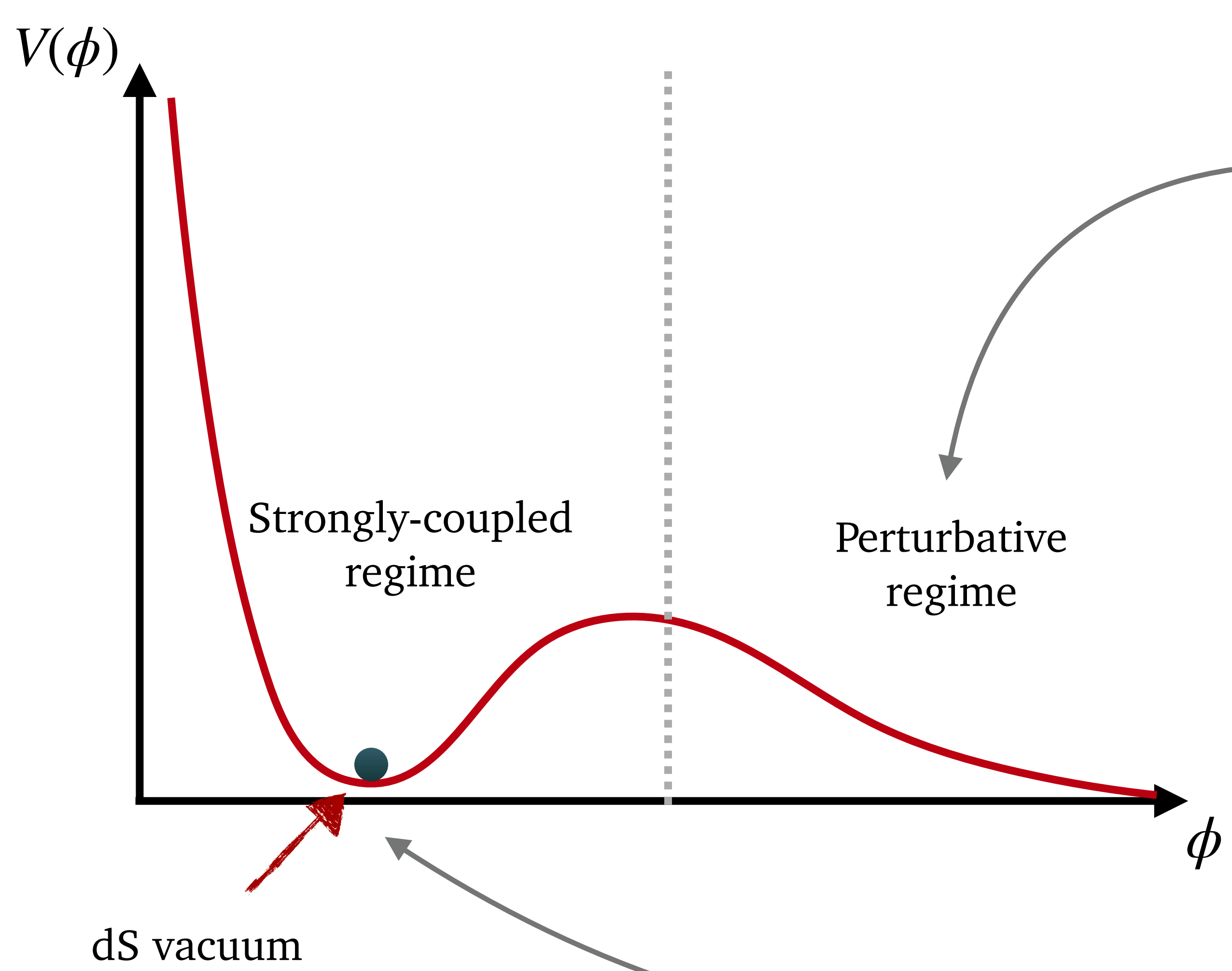
DE SITTER VACUA IN STRING THEORY

- Dine-Seiberg problem:

de Sitter vacua in string theory only at strong coupling

$\rightarrow$ *limited control over non-perturbative string theory!*



- at first order in coupling/moduli: runaway to Minkowski
- need higher order effects to generate de Sitter vacuum
  - ➡ strong coupling / interior of moduli space

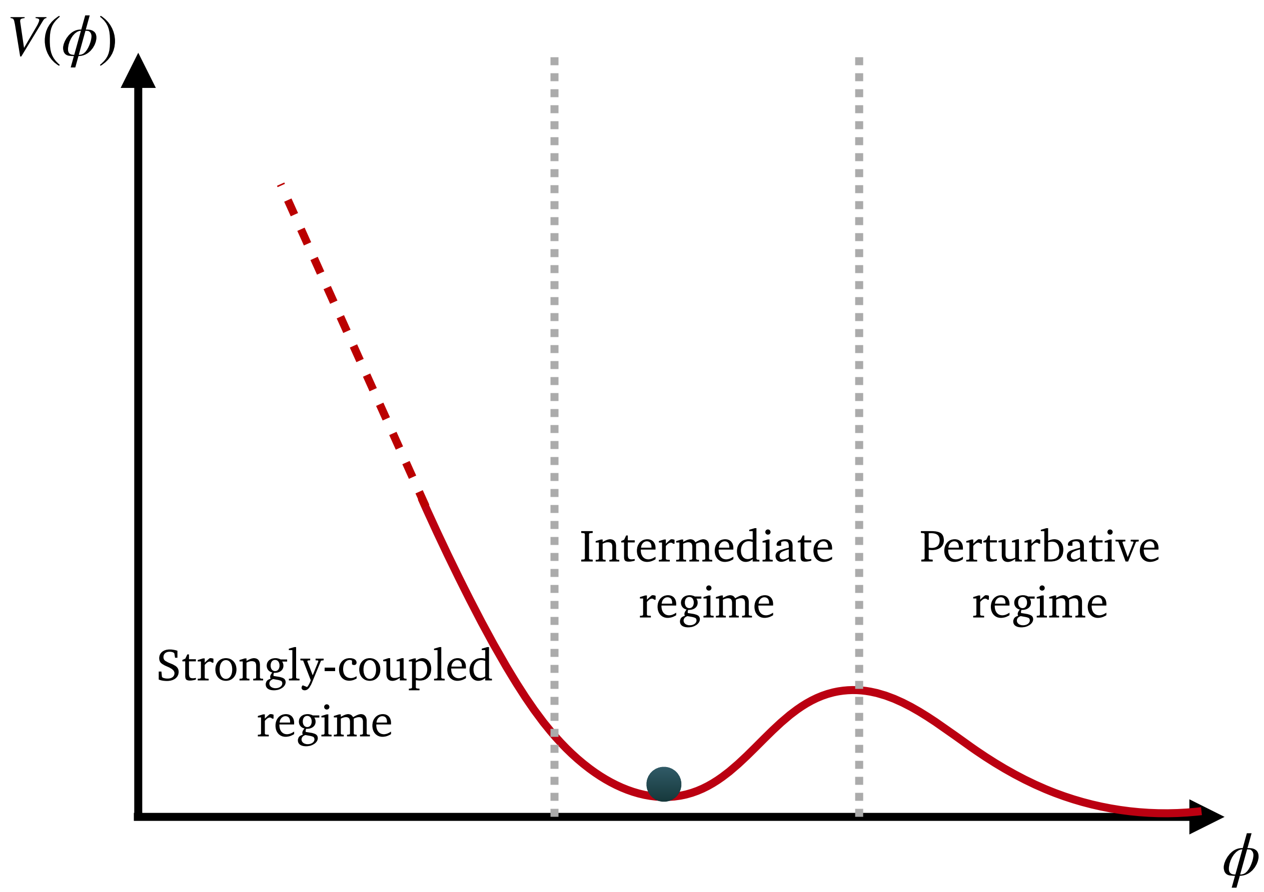
[Dine, Seiberg '85]

DE SITTER VACUA IN STRING THEORY

- Goal:

find de Sitter in intermediate regime

$\longrightarrow$ *need extra ingredients to avoid Dine-Seiberg*



Multi-step procedure:

1. Find a well-controlled AdS vacuum
2. Uplift the vacuum energy by a small perturbation

[Kachru, Kallosh, Linde, Trivedi '03; Balasubramanian, Berglund, Conlon, Quevedo '05]

DE SITTER VACUA IN STRING THEORY

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find de Sitter in intermediate regime

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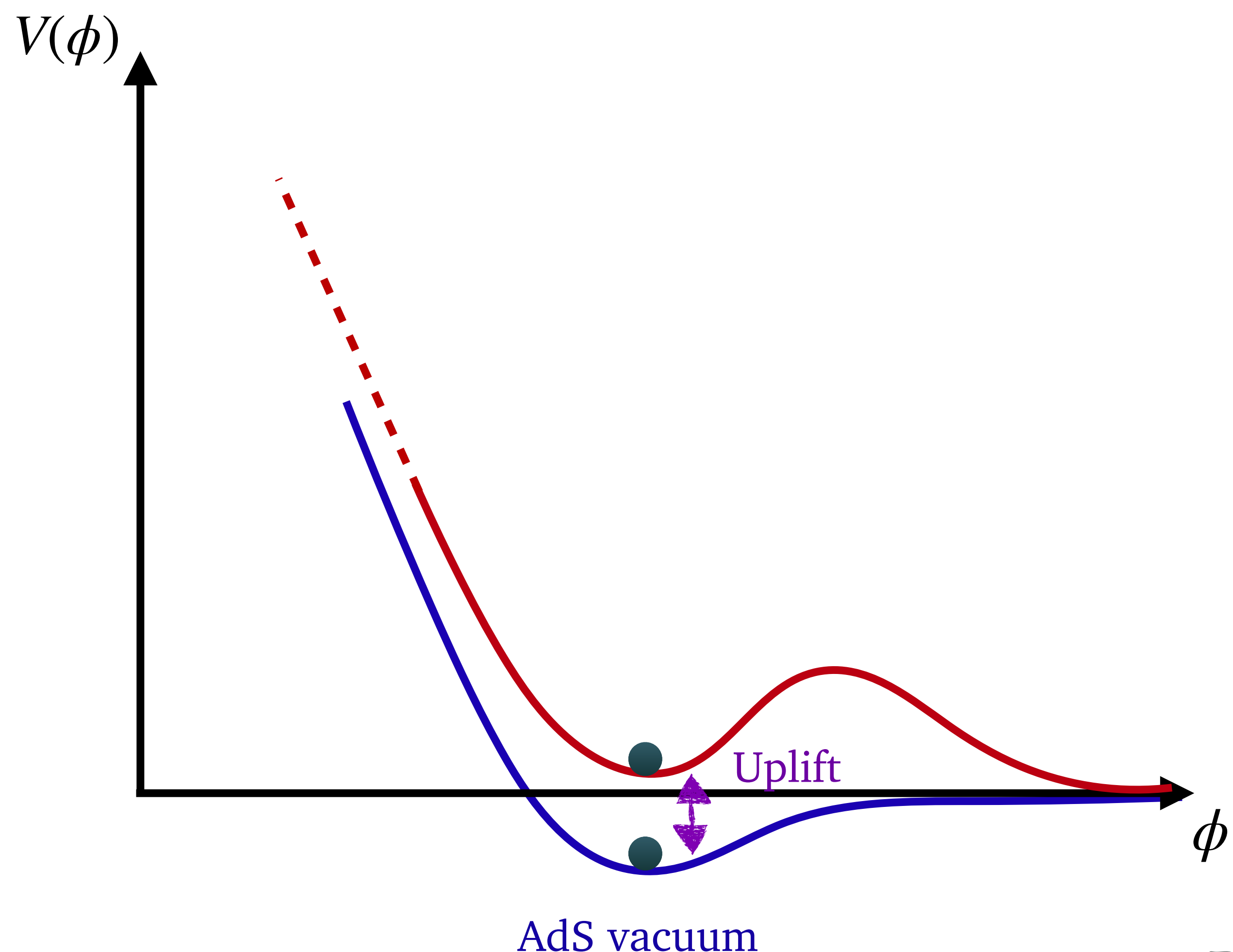

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CONTROLLED STRING THEORY DE SITTER VACUA

KKLT scenario: two step procedure:

This Talk!

1. stabilize moduli with fluxes + non-pert. effects

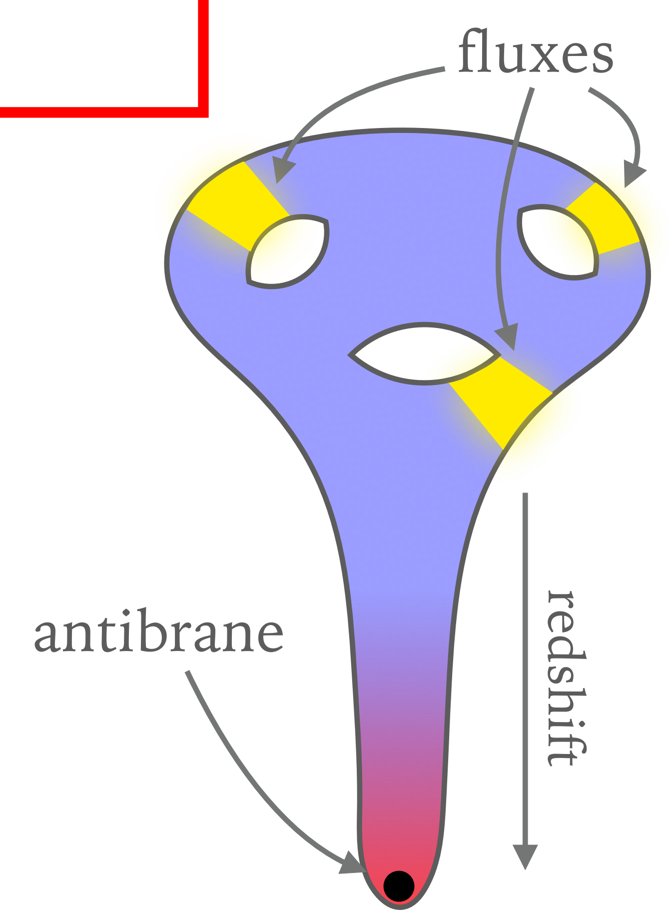
$\rightarrow$ SUSY vacuum with

$\Lambda < 0$ (AdS)

2. raise cosmological constant above zero
(perturb by antibrane in warped throat)

$\rightarrow$ broken SUSY and

$\Lambda > 0$



REVIEW:

KKLT ADS VACUA

COMPACTIFICATION ON CALABI-YAU MANIFOLDS

➤ IIB on Calabi-Yau (orientifold):

$$M_{10} = M_4 \times_w CY_3$$

➡ two classes of moduli:

- $h^{1,1}$ Kähler moduli (volumes of 2 or 4-cycles)
- $h^{2,1}$ complex structure moduli (volumes of 3-cycles)

VEV for 3-form fields $F_3, H_3 \neq 0$ on 3-cycles („fluxes“) $\longrightarrow$ fix size of 3-cycles $\longrightarrow$ masses for complex structure moduli

MODULI STABILIZATION

- tree-level $\mathcal{N} = 1$ superpotential:

$$W = \int G_3 \wedge \Omega$$

Does not depend on Kähler moduli!

$$(G_3 = F_3 - \tau H_3)$$

- F-term conditions

complex structure + axio-dilaton:

$$D_i W = 0$$

$$\Rightarrow \star G_3 = i G_3$$

Kähler moduli:

$$D_\alpha W \sim W$$

$\Rightarrow$ SUSY broken unless $W = 0$

- Resolution: non-perturbative quantum corrections:

Kähler moduli

$$W = \int G_3 \wedge \Omega + \sum_{\mathbf{k}} \mathscr{A}_{\mathbf{k}}(z^i, G_3)\ e^{-2\pi k^\alpha T_\alpha}$$

$\rightarrow$ W depends on Kähler moduli!

KKLT: ADS VACUUM

[Kachru, Kallosh, Linde, Trivedi ’03]

- Full scalar potential: $V = e^K \left(g^{a\bar{b}} D_a W \bar{D}_{\bar{b}} \bar{W} - 3\,|W|^2 \right)$

- Supersymmetry conditions:

  (for all moduli) $$D_a W = 0$$

Notice:

Existence of classical flux vacuum does not guarantee existence of KKLT AdS vacuum!

- Supersymmetric AdS vacuum:

$$\Lambda_{AdS} = \langle V \rangle = -3 \left(e^K |W|^2 \right) \Big|_{D_a W = 0}$$

control over instanton expansion:

$$\left| e^{-2\pi k^\alpha T_\alpha} \right| \ll 1$$

$\longrightarrow$

controlled uplift to dS only possible if

$$\Lambda_{AdS} \ll 1$$

M-THEORY / F-THEORY

- M-theory on Calabi-Yau 4-fold (from 11D to 3D)

$h^{3,1}$ complex str. moduli (volumes of 4-cycles) $\longrightarrow$ stabilized by $G_4 \neq 0$ (4-form flux)

- Classical superpotential [Gukov, Vafa, Witten '99]:

$$W \sim \int_{CY_4} G_4 \wedge \Omega$$

- Non perturbative corrections:

$$W = \int G_4 \wedge \Omega + \sum_{\mathbf{k}} \mathscr{A}_{\mathbf{k}}(z^i, G_4)\ e^{-2\pi k^\alpha T_\alpha}$$

M-THEORY / F-THEORY

- classical flux vacua:

$$D_i W = \int G_4 \wedge \chi_i = 0 \qquad \Rightarrow \qquad \begin{array}{c} G_4 = \star\, G_4 \\ G_4 \in H^{4,0} \oplus H^{2,2}_{+} \oplus H^{0,4} \end{array}$$

- KKLT like AdS vacua:

balance

- $G_4^{4,0} \neq 0$
- non-pert. corrections

$$\langle V \rangle = -4 \left(e^K |W|^2 \right) \Big|_{D_a W = 0} < 0$$

- Tadpole cancellation [Becker, Becker ’96; Sethi, Vafa, Witten ’96]:

$$N_{M2} + \frac{1}{2} \int_{CY_4} G_4 \wedge G_4 = \frac{\chi(CY_4)}{24}$$

$\chi(CY_4)$: Euler number of CY 4-fold

(F-theory: encodes IIB D7-charges)

HOLOGRAPHY AND KKLT

HOLOGRAPHIC PERSPECTIVE

- first step of KKLT requires

AdS vacuum with

$$\Lambda_{AdS} \ll 1$$

(consistency of EFT: $\Lambda_{AdS} \ll m_{KK}^2 \rightarrow$ scale separation)

- Is there a holographically dual CFT?

AdS/CFT duality:

$$\left|\Lambda_{AdS}\right| \sim \frac{1}{c}$$

central charge

(# degrees of freedom of CFT)

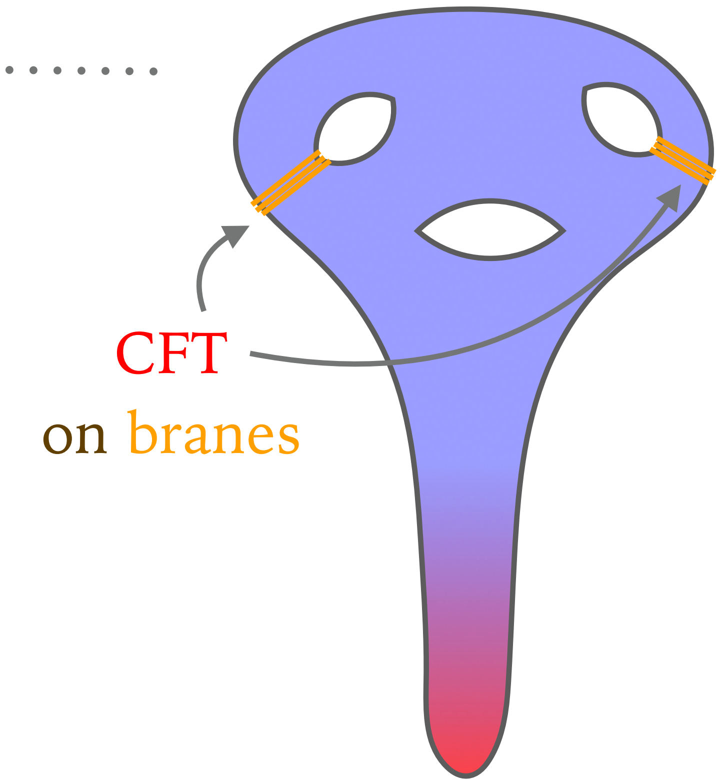
➡ Dual CFT must have $c \gg 1$

[Silverstein '03]
[Kounnas, Lüst, Petropoulos, Tsimpis '07]
[SL, Vafa, Wiesner, Xu '22]

HOLOGRAPHICALLY DUAL BRANE THEORIES

Conventional Holography:

Dualize Fluxes into Branes ⟶ *brane world volume theory flows to CFT*



- IIB with G_3 flux (KKLT):

D5 & NS5 branes ⟶ on special Lagrangian 3-cycles of CY_3 ⟶ SCFT in $d = 3, \mathcal{N} = 1$

- M-theory with G_4 flux (F-theory):

M5 branes ⟶ on special Lagrangian 4-cycles of CY_4 ⟶ SCFT in $d = 2, \mathcal{N} = (1,1)$

FLUXES AND DUAL BRANES

[Gukov, Vafa, Witten '99]

- M5-branes on 4-cycle: G_4 flux (+ #M2-branes) jumps
- Consider two different flux vacua with $G_4 = G_A$ and $G_4 = G_B$ on the same CY (+ different numbers of M2-branes).
- M5 brane on 4-cycle C dual to $\left[G_A - G_B\right]$

$\rightarrow$ *domain wall interpolating between the two vacua*

Tension: $$T \sim \int_C \Omega = \int \Omega \wedge (G_A - G_B)$$

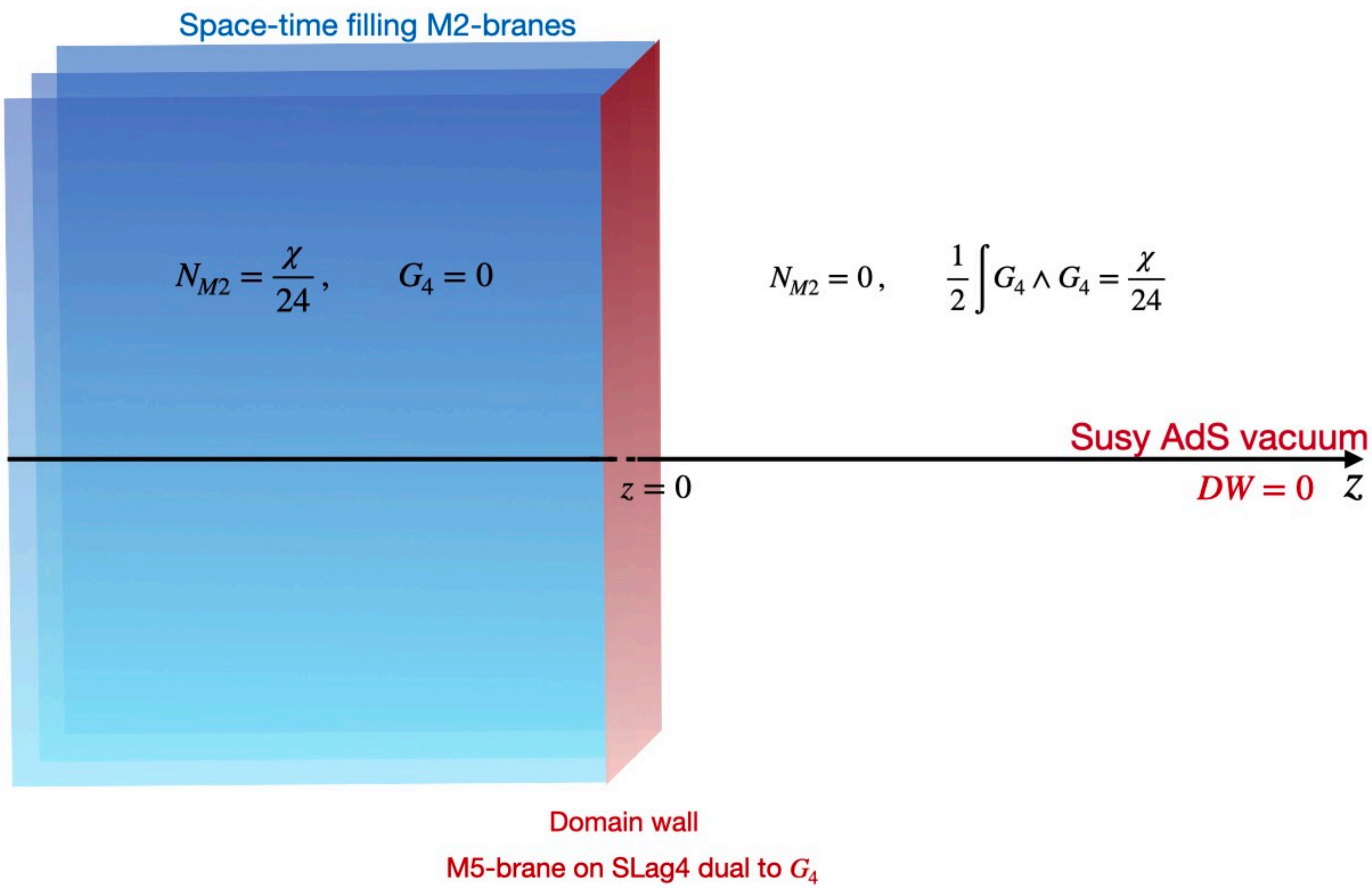
- Supergravity: Crossing of domain wall $\rightarrow$ W jumps:

$$\Delta W \sim T \qquad \Rightarrow \qquad W \sim \int \Omega \wedge G_4$$

GENERATING M5 BRANES

Extremal case: $G_B = 0$

$\rightarrow$ M5-brane on cycle dual to G_A:



BPS DOMAIN WALLS IN 3D SUPERGRAVITY

[Cvetic, Griffies, Rey '92]

see also: [Bandos, Farakos, Lanza, Martucci, Sorokin '18]
[Lanza, Marchesano, Martucci, Valenzuela '20]

- Metric ansatz:

$$ds^2 = e^{2D(z)}(-dt^2 + dx^2) + dz^2$$

- Tension:

SUGRA central charge: $$|Z| = e^{K/2}|W|$$

- attractor flow equations:

$$\frac{dD}{dz} = -\zeta|Z|$$

$$\zeta = \pm 1$$

$$\frac{d\phi^a}{dz} = 2\zeta g^{a\bar{b}}\partial_{\bar{b}}|Z|$$

(compare with BPS black hole attractors!)

[Kallosh '05]

End point of the flow ($z \to \pm\infty$):

$$\partial_a|Z| = 0 \quad \Rightarrow \quad D_a W = 0$$

$\rightarrow$ asymptotic AdS vacuum

SLAG CYCLES AND SELF DUAL FLUXES

- M5 branes: only supersymmetric on calibrated cycles!

classical calibration conditions:

$$J\Big|_C = 0 \qquad \text{"Lagrangian"}$$

$$\mathrm{Im}\left(e^{i\alpha}\Omega\right)\Big|_C = 0 \qquad \text{"special"}$$

$\rightarrow$ *special Lagrangian "SLag"*

NB: "special Lagrangian":
property of submanifolds, i.e. cycle representatives!

- Slag representatives exist if

$$|Z| = \frac{\left|\int_C \Omega\right|}{\left(\int \Omega \wedge \overline{\Omega}\right)^{\frac{1}{2}}} \qquad \text{has an extremum}$$

SLAG CYCLES AND SELF DUAL FLUXES (2)

- attractor flow and F-term conditions:

$$\partial_a |Z| = 0 \quad \overset{Z \neq 0}{\Leftrightarrow} \quad D_a W = 0$$

SLag representatives: critical point is not enough!

need extremum of $|Z|$!

F-term minimum: critical point sufficient

→ not every choice of self-dual fluxes is dual to a SLag cycle!

Example: "perturbatively flat" vacua by [Demirtas, Kim, McAllister, Moritz '19]

TOPOLOGICAL COUNT OF DEGREES OF FREEDOM

COUNTING DEGREES OF FREEDOM

[SL, Vafa, Wiesner, Xu '22]

- Brane world volume theory

UV: *supersymmetric but in general no CFT* $\xrightarrow{\textit{RG-flow}}$ *IR*: *supersymmetric CFT*

- c-theorem:

$$c_{UV} > c_{IR}$$

- AdS/CFT:

$$c_{IR} \sim \left|\Lambda_{AdS}\right|^{-1}$$

- Goal: Count degrees of freedom in UV

Determine c_{UV} for M5 branes on special Lagrangian cycle $\longrightarrow$ *lower bound on Λ_{AdS}*

COUNTING DEGREES OF FREEDOM (2)

[SL, Vafa, Wiesner, Xu '22]

- M5 branes on special Lagrangian cycle L_4

parametric growth: $L_4 \to NL_4$ ⟶ *How does c_{UV} scale with N?*

- M5 brane world volume theory:

reduction of six-dim. $\mathcal{N} = (2{,}0)$ theory on L_4 ⟶ two-dimensional $\mathcal{N} = (1{,}1)$ theory

χ: Euler number

count moduli of L_4 $\Rightarrow$ $$c_{UV} = \frac{3}{2}(2 + b_2^+ + b_2^- + 2b_1) = \frac{3}{2}\left(\chi(L_4) + 4b_1\right)$$

b_n: Betti numbers of L_4

COUNTING DEGREES OF FREEDOM (3)

[SL, Vafa, Wiesner, Xu '22]

- UV central charge:

$$c_{UV} = \frac{3}{2}\left(\chi(L_4) + 4b_1\right)$$

- Self intersection of L_4: $\chi(L_4) = L_4 \cdot L_4$

$$c_{UV} = \frac{3}{2}\left(L_4 \cdot L_4 + 4b_1\right) \quad \Rightarrow \quad c_{UV} \propto N^2$$

for $L_4 \to NL_4$

- Relation with Tadpole Condition:

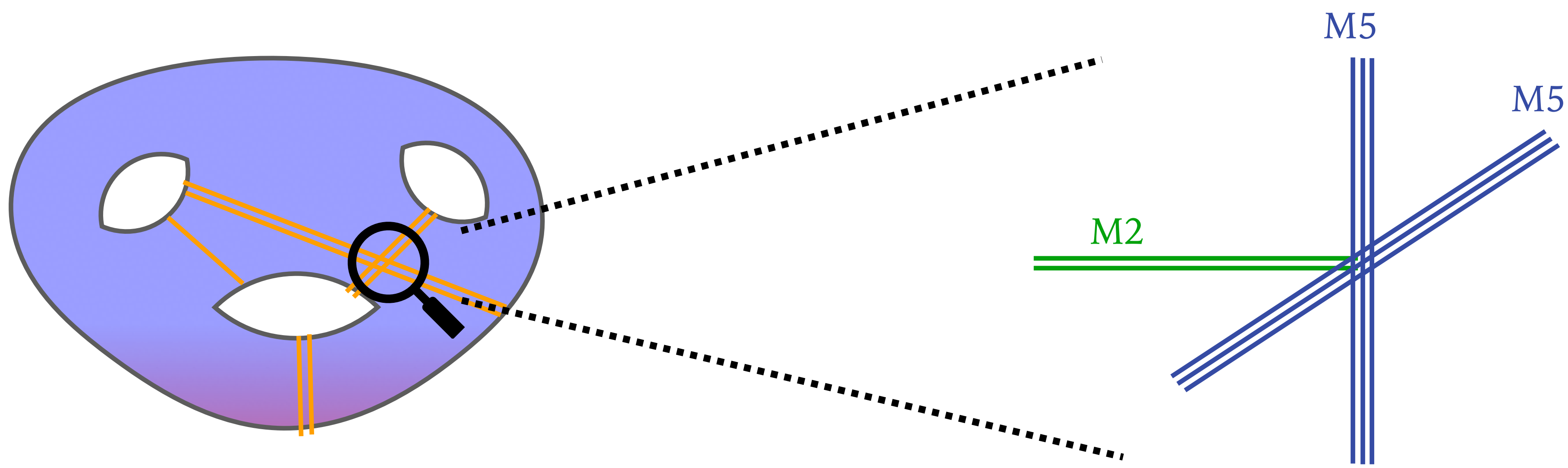
$$\frac{1}{2} L_4 \cdot L_4 = \frac{1}{2}\int_{CY_4} G_4 \wedge G_4 \leq \frac{\chi(CY_4)}{24} \quad \Rightarrow \quad \boxed{c_{IR} \lesssim \beta\, \chi(CY_4)}$$

DEGREES OF FREEDOM IN SUPERGRAVITY

COUNTING OF DEGREES OF FREEDOM IN SUPERGRAVITY

[Bena, Li, SL '24]

- Most degrees of freedom are localized at brane intersections:



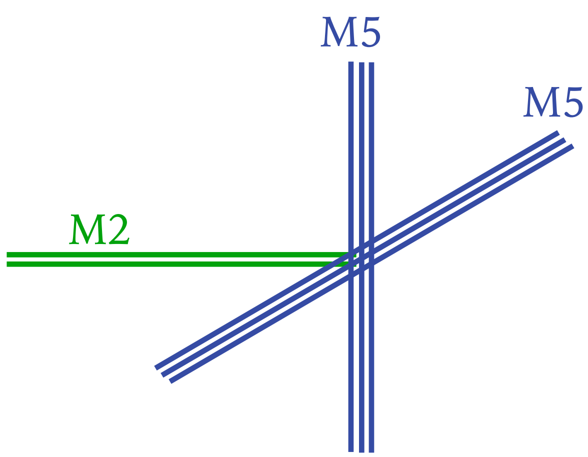
- Local description: Intersecting branes in flat space

- Goal: Find supergravity solution and count degrees of freedom holographically

INTERSECTING BRANES IN SUPERGRAVITY

- Local brane setup:

	t	y	z	1	2	3	4	5	6	7	8
M5	x	x		x	x	x	x				
M5	x	x						x	x	x	x
M2	x	x	x								



- Supergravity solution: [de Boer, Pasquinucci, Skenderis '99]

$$ds^2_{11} = \left(H_T^{-1} Z_5 Z_5'\right)^{2/3}\left[\left(Z_5 Z_5'\right)^{-1}(-dt^2 + dy^2) + dz^2\right]$$

$$+H_T^{1/3}\left(Z_5\right)^{-1/3}\left(Z_5'\right)^{2/3}\left[dr^2 + r^2 d\Omega^2_{(1)}\right] + H_T^{1/3}\left(Z_5\right)^{2/3}\left(Z_5'\right)^{-1/3}\left[dr'^2 + r'^2 d\Omega^2_{(2)}\right]$$

Harmonic functions:

$$Z_5 = 1 + \frac{n_F}{r'^2} \qquad Z_5' = 1 + \frac{n_F'}{r^2}$$

$$H_T = \left(1 + \frac{n_T}{r'^2}\right)\left(1 + \frac{n_T}{r^2}\right)$$

NEAR-BRANE GEOMETRY

- In the near-horizon limit $r, r' \to 0$: [de Boer, Pasquinucci, Skenderis '99]

$$ds_{11}^2 = \underbrace{\left(\frac{u}{l_{\mathrm{AdS}}}\right)(-dt^2 + dy^2) + \left(\frac{l_{\mathrm{AdS}}}{u^2}\right)^2 du^2}_{\mathrm{AdS}_3} + \underbrace{\left(n_T n_F\right)^{\frac{1}{3}} \left(d\Omega^2 + d\Omega'^2\right)}_{S^3 \times S^3} + \underbrace{d\lambda^2 + \left(\frac{n_F^2}{n_T}\right)^{\frac{2}{3}} dz^2}_{\mathbb{R}^2}$$

with $\quad l_{\mathrm{AdS}} = \frac{1}{\sqrt{2}} \left(n_T n_F\right)^{\frac{1}{6}}$

- Normalization of brane charges / densities: [Bena, Li, SL '24]

n_T, n_F:
brane densities

$\longrightarrow$

compactify formally
$\mathbb{R}^2 \to S_\lambda \times S_z$
with radii: R_λ, R_z
(later: $R_\lambda, R_z \to \infty$*)*

HOLOGRAPHIC CENTRAL CHARGE

[Bena, Li, SL '24]

- Normalization of brane charges:

$$N_5 = \frac{1}{(2\pi)^3} \int_{S_z \times S^3} G_4 = R_z\, n_F$$

$$N_2 = \frac{1}{(2\pi)^6} \int_{S_\lambda \times S^3 \times S^3} \left(\star\, G_4 - \frac{1}{2} C_3 \wedge G_4 \right) = \frac{\sqrt{2}}{4\pi} R_\lambda \left(n_T n_F\right)^{\frac{5}{6}}$$

- Central charge of dual CFT_2:

$$l_{\mathrm{AdS}_3} = \frac{1}{\sqrt{2}} \left(n_T n_F\right)^{\frac{1}{6}}$$

$$c = \frac{3}{2} \frac{l_{\mathrm{AdS}_3}}{G_{\mathrm{N}}^{(3)}} = 3 N_2 N_5$$

$$\frac{1}{G_{\mathrm{N}}^{(3)}} = \mathrm{vol}\left(S^3 \times S^3 \times S_\lambda \times S_z\right) \sim R_z R_\lambda \left(n_T n_F\right)^{\frac{2}{3}} n_T$$

INTERSECTING BRANES IN CY4

[Bena, Li, SL '24]

- Highest central charge: all branes intersect in one point:

Tadpole cancellation:

$$N_2 = \frac{\chi(CY_4)}{24} \sim N_5^2 \quad \longrightarrow \quad c \sim N_2 N_5 \sim N_5^3$$

- However: Generic situation: branes intersect at many points:

each intersection: $c \sim \mathcal{O}(1)$ $\times$ *#intersections:* N_5^2 $\longrightarrow$ $c \sim N_5^2 \sim \chi(CY_4)$

IIB: INTERSECTING D5 / NS5 BRANES ON CY3-ORIENTIFOLDS

[Bena, Li, SL '24]

- Local brane setup:

	t	x	y	z	1	2	3	4	5	6
D5	x	x	x		x	x	x			
NS5	x	x	x					x	x	x
D3	x	x	x	x						

- Near brane geometry:

[D'Hoker, Estes, Gutperle '07 (2x)]
[Aharony, Berdichevsky, Berkooz, Shamir '11]
[Assel, Bachas, Estes, Gomis '11]

$$AdS_4 \times_w S^2 \times S^2 \times \Sigma_2$$

Riemann surface

- Scaling of central charge:

$$c \sim \frac{l_{AdS_4}}{G_N^{(4)}} \sim \left(N_{D5} N_{NS5}\right)^2$$

CONSEQUENCES FOR KKLT

CONSEQUENCES

[SL, Vafa, Wiesner, Xu ’22]

- central charge bounded by tadpole condition:

$$\Lambda_{AdS} \sim \frac{1}{c} \gtrsim \frac{1}{\chi(CY_4)}$$

Euler number:

$$\chi(CY_4) = 6(8 + h^{1,1} + h^{3,1} - h^{2,1})$$

(see also “tadpole conjecture” [Bena, Blåbäck, Graña, SL ’20])

- effective cutoff in gravitational theories with N light d.o.f.

[Dvali ’07]

species scale: $$\Lambda_{\text{species}} \sim \frac{1}{N}$$

Here:

$$N \sim \chi(CY_4)$$

$$\Rightarrow \quad \boxed{\Lambda_{AdS} \gtrsim \Lambda_{\text{species}}}$$

AdS scale is above cutoff of the EFT!

→ no sensible EFT description!

(see also [Blumenhagen, Gligovic, Kaddachi ’22])

CONCLUSIONS

- Holographic dual to KKLT AdS vacua:

world-volume SCFT of
M5 (M-theory) / D5-NS5 (IIB) branes

- Holography:

counting d.o.f. of M5-brane CFT:

$$|\Lambda_{AdS}| > \left(N_{moduli}\right)^{-1}$$

- comparison with species scale:

$$\Lambda_{AdS} \gtrsim \Lambda_{EFT} \longrightarrow$$

no scale separated KKLT like AdS vacua

THANK YOU!