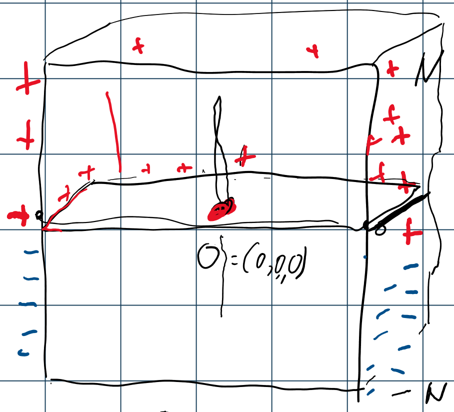
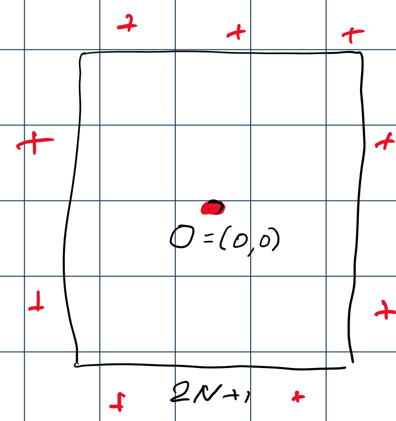
No roughening

Wednesday, January 11, 2023 11:59





$D=3$ $D=2$

Th $\boxed{\langle \sigma_{(0,0,0)} \rangle^{\beta}_{\pm, 2N+1} \geq \langle \sigma'_{(0,0)} \rangle^{\beta}_{+, 2N+1}} > 0$

$(0,0,0) \longrightarrow (0,0,-1)$ $\quad \underline{\langle \sigma_{(0,0,-1)} \rangle^{\beta}_{\pm, 2N+1}} \leq \langle \sigma'_{(0,0)} \rangle^{\beta}_{-, 2N+1}$

$\wedge$

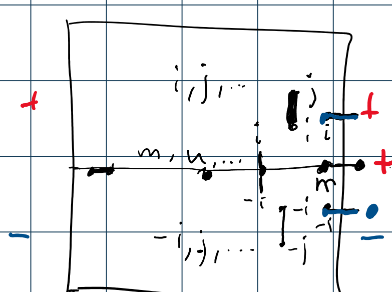
0

at very low $T \to$ $\quad \langle \sigma'_{(0,0)} \rangle^{\beta = \frac{1}{T}}_{+}$ is close to $+1$

$\langle - \text{''} - \rangle_{-}$ $\quad$ — '' — $\quad -1$

Proof. We compare 3D system with 2D system.

$\sigma \qquad \sigma'$



m, n – 2D indexes

$$H^{(3)}(\sigma) = \Big(\underbrace{\sum_{\substack{i,j \\ i \sim j}} \underline{(\sigma_i \sigma_j + \sigma_{-i}\sigma_{-j})}}_{\mathrm{I}} + \underline{\sum_{\substack{m,n \\ m \sim n}} \sigma_m \sigma_n} + \underline{\sum_{\substack{i,m \\ i \sim m}} \sigma_m (\sigma_i + \sigma_{-i})} +$$

$$\sum_{i\sim j} \quad \sum_{m\sim n} \quad \sum_{\substack{i,m \\ i\sim m}}$$

$$\underbrace{+\sum_i h_i (\sigma_i - \sigma_{-i})}_{\text{b. conditions}} + \sum_m H_m \sigma_m$$

$$H^{(2)}_{(\sigma')} = \sum \sigma'_m \sigma'_n + \sum_m H_m \sigma'_m$$

$$\alpha_i = \frac{1}{2}(\sigma_i + \sigma_{-i}) \qquad \alpha_m = \frac{1}{2}(\sigma_m + \sigma'_m)$$

$$\beta_i = \frac{1}{2}(\sigma_i - \sigma_{-i}) \qquad \beta_m = \frac{1}{2}(\sigma_m - \sigma'_m)$$

$$(\sigma^{(3)} \cup (\sigma')^{(2)}) \longrightarrow H^{(3)}(\sigma^{(3)}) + H^{(2)}((\sigma')^{(2)}) =$$

$$= \Big(\sum_{i\sim j} (\alpha_i \alpha_j + \beta_i \beta_j) + \sum_{m\sim n} (\alpha_m \alpha_n + \beta_m \beta_n) +$$

$$\underbrace{+ \sum_{i\sim m} \alpha_i (\alpha_m + \beta_m) + \sum_i h_i \beta_i + \sum_m H_m \alpha_m}_{\text{all monomials have positive coeff.}}$$

i, m, $-i$

$$\int \alpha_i^{k_i} \beta_i^{l_i} \, d\sigma \, d\sigma' \geq 0$$

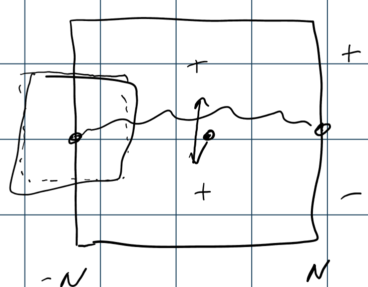
$$= \left\langle \prod_i \alpha_i \prod_j \beta_j \right\rangle \geq 0$$

In part., $\langle \beta_0 \rangle > 0$; but

$$\beta_{(0,0,0)} = \underline{\sigma_{(0,0,0)} - \sigma'_{(0,0)}} \implies \underline{\langle \sigma^{(3)}_{(000)} \rangle^{\beta}_{\pm} \geq \langle \sigma^{(2)}_{0,0} \rangle^{\beta}_{+}} .$$

In other words; $T^{(3D)}_{\text{rough}} \geq T^{(2D)}_{\text{crit.}}$ ■

Back to 2D:



fluct. are of $\sqrt{N}$.

Gibbs states of the Ising in the halfplane.