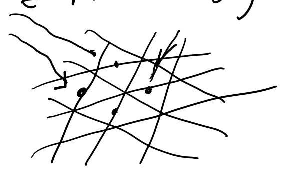
# Equicontinuity. End of the proof of conformal invariance.

Friday, December 9, 2022 11:11

$\delta \rightarrow \underline{\underline{F_\delta(z)}}$ $\quad$ $\underline{z \in \mathcal{O}}$ $\qquad$ $F_\delta(z) = \underline{F_a(z)} + j\,\underline{F_b(z)} + j^2\,\underline{F_c(z)}$

well defined

$z \in \mathrm{Mid}(\mathcal{O}_\delta)$





$\mathcal{O}_\delta$

<u>Wish</u>: $\underline{F_\delta(z) \xrightarrow[\delta \to 0]{} F(z)\ ; \quad ??}$

<u>Th.</u> $\lim\limits_{\delta \to 0} F_\delta$ – exists.

<u>When</u> a family $\underline{\text{of fn. is compact ?}}$

<u>Arcela</u> th. $\quad$ ① $\underline{\text{Uniformly bdd.}}$

② $\underline{\text{Equicontinuous.}}$

$f_\alpha \in \mathcal{F}$ – family of fn.

$f$ – cont. at $x$ $\Longleftarrow \left\{ \forall \varepsilon \ \exists\, \delta(f, \varepsilon) \text{ s. that } \forall f \right.$

$\left. \text{if } |y-x| < \varepsilon \Longrightarrow |f(y)-f(x)| < \delta \right\}$

$\underline{\mathcal{F} - \text{equicontinuous}}$

$\Longleftarrow \left\{ \underline{\forall \varepsilon} \quad \underline{\exists\, \delta(\varepsilon)} \quad \forall f_\alpha \quad \text{———} \ '' \ \text{———} \right.$

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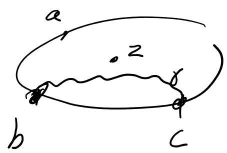
$F^\delta(z)$ – compact $\quad \underline{\delta \to 0}$

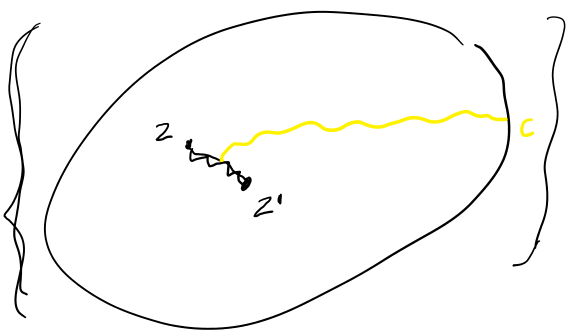
$F^{\delta}(z)$ — compactness in $\delta$   $\delta \to 0$




$$F^{\delta}_{a}(z) = \mathbb{P}\mathrm{r}_{1/2}\big(\gamma:\ b \longleftrightarrow c;\ \gamma \text{ goes below } z\big\}$$


$$\left|F^{\delta}_{a}(z) - F^{\delta}_{a}(z')\right| \leq$$

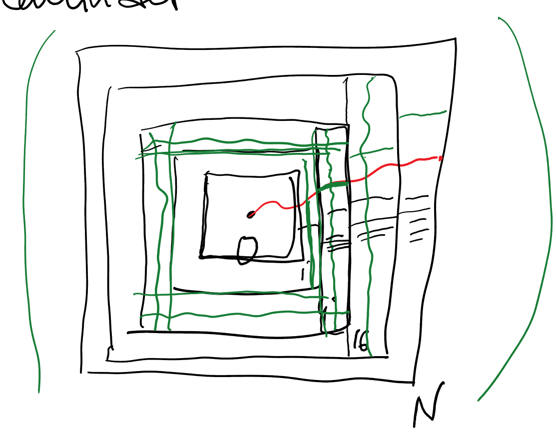
$$\leq \mathbb{P}_{1/2}\big(\gamma:\ a \longleftrightarrow b:\ \gamma - \text{separates } z \text{ and } z'\big)$$

$P_{1/2}$ 

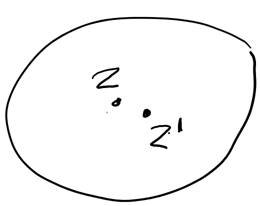

$\longrightarrow 0$

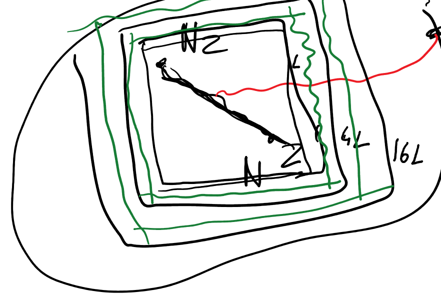
as $|z - z'| \longrightarrow 0$ (uniformly in $\delta$)

Reminder   $p = 1/2$

$\mathbb{P}\mathrm{r}_{1/2}$ 


$$\leq (c)^{\log_4 N} = N^{-\beta}$$




$\mathcal{O}$ $\longrightarrow$ $\mathbb{P}\mathrm{r}_{1/2}$ 


NO

$$\leq \; c^{\log_4 N - \log_4 |z-z'|N} = |z-z'|^{\beta}$$

$\blacksquare$