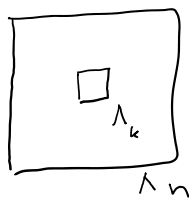


① If $p > p_{cr}$

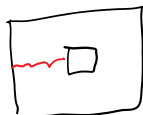
$n \gg k \gg 1$



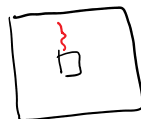
$$P_p \left(\text{red line in } \Lambda_n \right) \xrightarrow{n \rightarrow \infty} 1$$



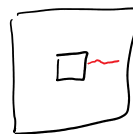
A_1



A_2



A_3



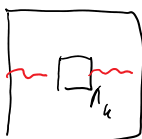
A_4

$$P_p(A_i) \geq 1 - \left[1 - P_p(A_1 \cup A_2 \cup A_3 \cup A_4) \right]^{1/4}$$

$$P_p(\Lambda_k \leftrightarrow \partial \Lambda_n) \xrightarrow{n \rightarrow \infty} 0$$

$$P_p(\Lambda_k \leftrightarrow \infty) \xrightarrow{\text{as } k \rightarrow \infty} 0$$

$$P_p(A_2 \cap A_4) \geq$$

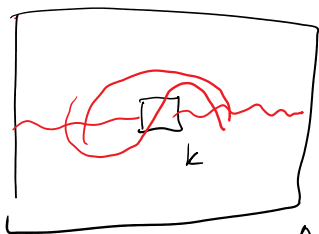


Λ_n

$$1 - 2 P_p(\Lambda_k \leftrightarrow \partial \Lambda_n)$$

$$P(C) \geq 1 - c \quad P(D) \geq 1 - d$$

$$\Rightarrow P(C \cap D) \geq 1 - c - d$$



Λ_n

No connection left-right $\Rightarrow$

two inf clusters. — But

— this is impossible $\Rightarrow$

∃ connection:



② $p < p_{cr} \Rightarrow$

$$P_p \left\{ \text{red line in } \Lambda_n \right\} \xrightarrow{n \rightarrow \infty} 0$$

⊗

$$\sum_{k=1}^n P_p \left\{ \text{red line in } \Lambda_k \right\}$$

$c_p > 0$

(*) $\sum_{k=1}^{\infty} P_p \left\{ \left[\begin{array}{c} \text{rectangle of size } k \times H_n \\ \text{with a crossing} \end{array} \right] \right\}$

$P_p \left\{ \left[\begin{array}{c} \text{rectangle of size } 2n \times 2n \\ \text{containing a } k \times H_n \text{ rectangle with a crossing} \end{array} \right] \right\} \leq \exp \{ -c_p n \}$

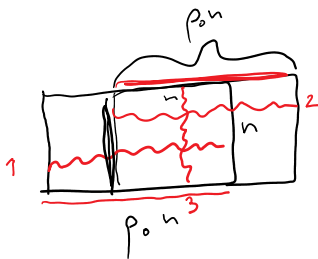
(*) $\sum_{k=1}^{\infty} \dots \leq n \exp \{ -c_p n \} \rightarrow 0$ (exp. fast) 1/13

RSW- theory (Russo-Seimour-Welsh)

$c(p)$ $\leq P_{1/2} \left\{ \left[\begin{array}{c} \text{rectangle of size } n \times pn \\ \text{with a crossing} \end{array} \right] \right\} \leq \underline{\underline{1 - c(p)}}$ * - do not depend on n ,

$\forall p \exists c(p) \in (0, 1/2)$

Proof. 1). If we know * for some $p_0 > 1$, then we know * for all p -s.



If $1 \cap 2 \cap 3$ - happens $\implies$ crossing in $(2p_0 - 1)n$ rectangle.

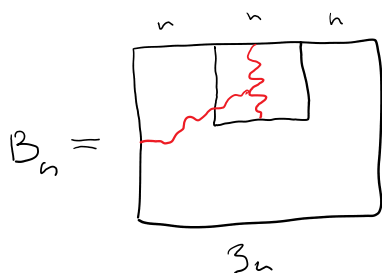
$$P_{1/2}(1 \cap 2 \cap 3) \geq \underbrace{P_{1/2}(1)}_{\geq \frac{1}{2}} \underbrace{P_{1/2}(2)}_{\geq \frac{1}{c(p_0)}} P_{1/2}(3)$$

$$\geq c(p_0) \cdot \frac{1}{2}$$

2). $P_{1/2} \left(\left[\begin{array}{c} \text{rectangle of size } 2n \times 3n \\ \text{with a crossing} \end{array} \right] \right) \geq \frac{1}{128 \binom{1}{2^7}}$

$$1/2 \mid \boxed{} \mid \quad \text{---} (\sim 2^7)$$

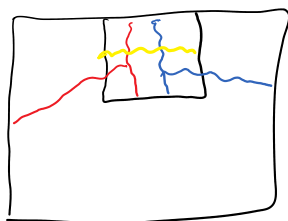
2a



$$\text{If } P_{1/2}(B_n) \geq c$$

$$\text{--- then } P_{1/2}(B_{3/2}) \geq c^2 \cdot \frac{1}{2}$$

Proof.



2b

$$P_{1/2}(B_n)$$

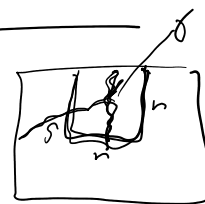
General fact:

$$\text{if } C_1, \dots, C_k, \quad C_i \cap C_j = \emptyset$$

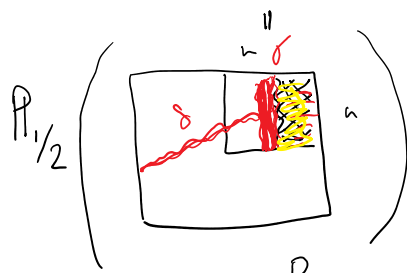
$$P(B) \geq \sum_{i=1}^k P(B \cap C_i)$$

$$P_{1/2}(B_n) \geq$$

$$\sum_{\gamma} P_{1/2} \left\{ \gamma; \begin{array}{l} \gamma - \text{connected by a path } \delta, \\ \gamma - \text{is the rightmost path (top/bottom) in } \square_n \end{array} \right\}$$



$$= \sum_{\gamma} P_{1/2}(\gamma) P_{1/2} \left(\exists \delta \text{ connecting } \gamma \text{ to the left side of } 2n \times 3n \mid \gamma \right)$$



because γ - is the rightmost connection.



$$\begin{aligned}
 & \mathbb{P}_{1/2} \left(\begin{array}{c} 2n \\ \text{diagram with red line and } 1/2 \text{ label} \\ 2n \end{array} \right) = \frac{1}{2} \left[\mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with } R_n \\ R_n \end{array} \right) + \mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with } \bar{R}_n \\ \bar{R}_n \end{array} \right) \right] \\
 & \mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with } \textcircled{IV} \\ \text{diagram with } \approx \end{array} \right) \geq \mathbb{P}_{1/2} \left(\begin{array}{c} 2n \\ \text{diagram with red line} \\ 2n \end{array} \right) = \frac{1}{2}
 \end{aligned}$$



Corollary.

Power-law decay of correlations.

$$\left| \frac{1}{2n} \leq \mathbb{P}_{1/2} \left[0 \longleftrightarrow \partial \Lambda_n \right] \leq n^{-\alpha} \right|$$

①

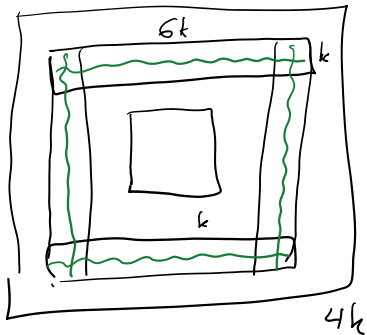
$$\frac{1}{2} = \mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with red line and } n \text{ label} \\ n \end{array} \right) \leq$$

$$\leq \sum_x \mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with red line and } x \text{ label} \\ 2n \end{array} \right) = n \mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with red line and } 0 \text{ label} \\ 2n \end{array} \right)$$

②

$$\mathbb{P}_{1/2} \left(\begin{array}{c} \text{diagram with red line and } n_k \text{ label} \\ n_k \end{array} \right) \leq \underline{\underline{1-c}} \quad \text{unif. in } k$$

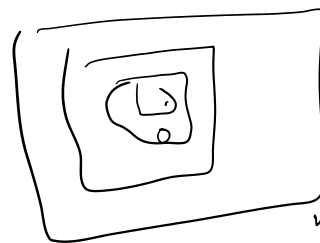
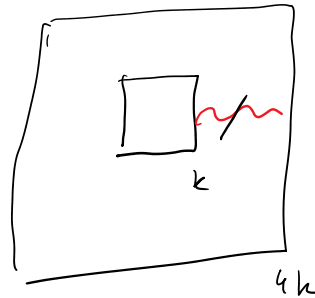
$$\frac{1}{2} \left(\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \right) =$$



$$\mathbb{P}_{1/2} \left(\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \right) > c_0$$

$\Rightarrow$ no red connection between n

$$\geq (c_0)^4$$



number of layers

is $\log_4 n$.

$$\left[(c_0)^4 \right]^{\log_4 n} = n^{-\alpha}$$

$$\underline{\alpha > 0.}$$