Ising model. Infinite interacting system.

$\mathbb{Z}^d$, $x \in \mathbb{Z}^d$, $x \longrightarrow \sigma_x = \pm 1$, $\Omega = \{\sigma\}$, $\sigma = \{\sigma_x, x \in \mathbb{Z}^d\}$

$\sigma \longrightarrow H(\sigma) = -\sum_{x \sim y} \sigma_x \sigma_y$ — hamiltonian (of (σ))

$\boxed{x, y \text{ — nearest neighbors, } |x-y| = 1}$

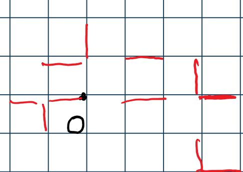
T — temperature, $\beta = \frac{1}{T}$ — inverse temp.;

$P(\sigma) \sim \exp\{-\beta H(\sigma)\}$ — prob. distr. on Ω.

Percolation. — infinite, no interaction.

on $\mathbb{Z}^2$ $\quad p \in [0, 1]$

bond ≡ edge = $(x, y) : |x-y| = 1$



open ≡ red — with prob. p.

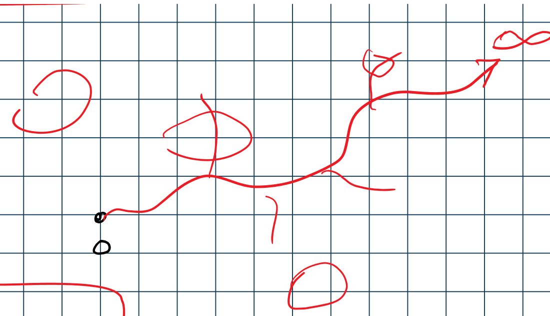
$e \longrightarrow \omega_e = \begin{cases} 1 & e \text{ — open} \\ 0 & e \text{ — closed.} \end{cases}$

configuration $= \{\omega_e, e \in E\}$

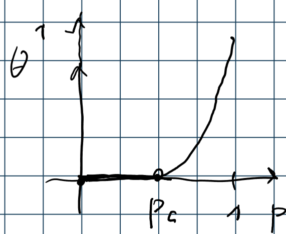
E — edges of $\mathbb{Z}^2$

$\boxed{\text{Can one percolate from } 0 \text{ to } \infty?}$

$0 \longrightarrow \infty$

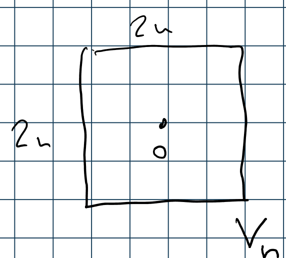
Percolation $\Longleftrightarrow$ 

$$P_p\{0 \to \infty\} = \theta(p)$$



$p_c(d=2) = 1/2$ (Kesten)

Th. 1. : $\theta(p) = 0$ for p small enough. $(d=2)$



$P_r \equiv P_p$ $\quad P_p\{$  $\}$

$$\leq \sum_{\gamma : 0 \to \partial V_n} P_p\{\gamma\text{-open}\} \leq$$

$P_p\{\gamma\text{-open}\} = p^{|\gamma|}$ $\quad |\gamma|$ – length of (γ).

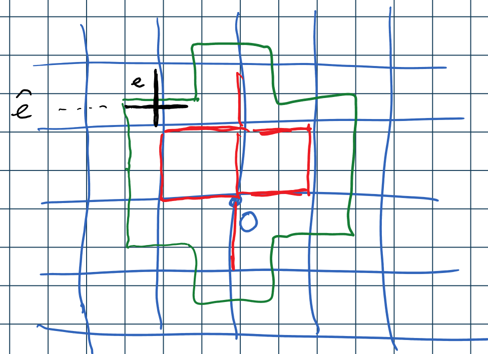
paths of length ℓ, starting at 0?

$$\leq 4 \cdot 3^{\ell-1} \sim 3^{\ell}$$

$$\leq \sum p^{\ell} 3^{\ell} \longrightarrow 0 \quad \text{if} \quad p < \frac{1}{3}$$

$$\leq \sum_{\ell > n} p^{\ell} \cdot 3^{\ell} \xrightarrow[n \to \infty]{} 0 \qquad \text{if} \quad p < \frac{1}{3} \qquad \blacksquare$$

<u>Th. 2</u> $\quad \theta(p) \to 1$ as $p \to 1$.



$$e \in E \longleftrightarrow \hat{e}$$

$$\omega_e \longrightarrow \hat{\omega}_{\hat{e}} = \begin{cases} 1 & \text{iff} \quad \omega_e = 0 \\ 0 & \text{iff} \quad \omega_e = 1 \end{cases}$$

$$P_p(0 \not\longleftrightarrow \infty) \leq \sum_{\gamma \text{ - circuits on } \hat{E}} P_p(\gamma)$$

$$\begin{array}{c} \text{||} \\ 1 - \theta(p) \end{array}$$

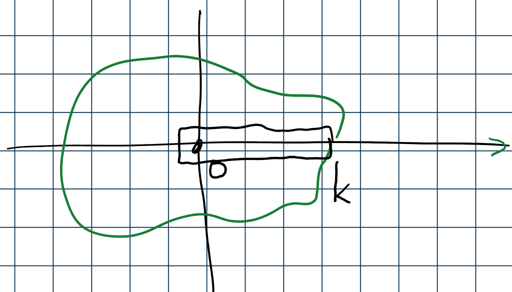
γ – encircling the origin, – open bonds of dual lattice

$$P(\gamma) = (1-p)^{|\gamma|}$$

$$= \sum_{\gamma} (1-p)^{|\gamma|} = \sum_{k} \sum_{\substack{\gamma: \\ k \in \gamma}} (1-p)^{|\gamma|} \leq \sum_{k \geq 1} \sum_{\substack{\ell: \\ \ell \geq 2k+2}} (1-p)^{\ell} \cdot 3^{\ell}$$

$$\Downarrow$$

$$|\gamma| \geq 2k+2$$

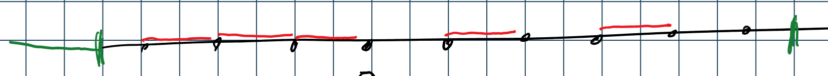


$$\sum_{k \geq 1} \sum_{\ell \geq 2k+2} 3^{\ell} (1-p)^{\ell} \longrightarrow 0 \quad \text{as } p \to 1$$

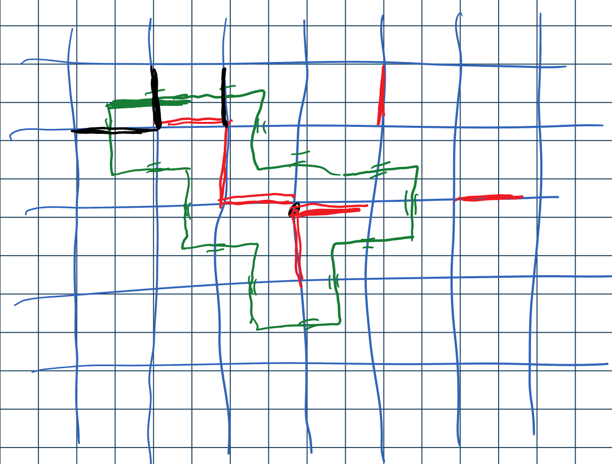
$\ell - 1$

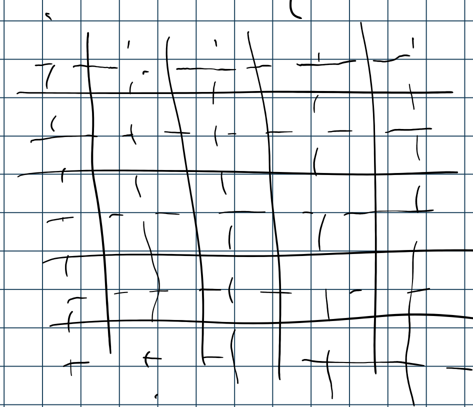
$$4 \cdot 3^{\ell - 1}$$

$\mathbb{Z}^1$

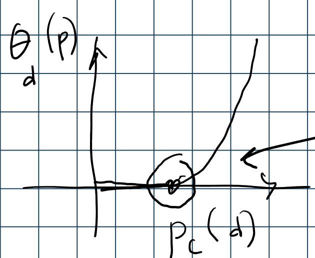


$$\theta(p) = \begin{cases} 1 & \text{if } p = 1 \\ 0 & \text{if } p < 1. \end{cases}$$





$\mathbb{Z}^d$ $\quad d \geq 3 \quad d \gg 3$



$d = 3$ Is not known: